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ALTERNATE-CURRENT CONDENSERS AND DIELECTRIC HYSTERESIS.[1]

By Frederick Bedell, N. F. Ballantyne, and R. B. Williamson.

WITHIN recent years the electromagnetic effects of alternating currents have been quite thoroughly studied, but the electrostatic phenomena have not received such careful attention, and many points of interest still remain to be investigated, among which may be mentioned that of hysteresis loss in the dielectric of a condenser. Not only from a purely physical point of view is it desirable to know as much as possible about the behavior of condensers, but also for practical reasons, inasmuch as condensers are likely to be used more extensively in the future in connection with alternating currents. For these reasons it was thought that the results of some experiments recently performed might be of interest. A description of what little was done may furnish some suggestions to those working along the same lines.

The condensers, six in number, upon which the experiments were performed are intended to be used commercially on 500-volt circuits. The plates are of tinfoil, the useful part of which is of the following dimensions: length, 10½ inches; width, 8 inches; thickness, .0007 inch. The dielectric is of waxed paper .0043 inch thick. There are sixty-five sheets of tinfoil (total)

[1] A paper read at the Madison Meeting of the American Association for the Advancement of Science, August, 1893.

in each slab, and two of these slabs are placed together in one tin case.

As one of the first things to be determined about a condenser is its capacity, this was measured for each of them by Thomson's method of mixtures, which is described in various laboratory manuals.

Another method used for finding the capacity was that of measuring the current in the condenser for any particular value of an harmonic electromotive force impressed at the terminals. Since the current in a circuit containing capacity alone when $e = E \sin \omega t$ is $I = CE\omega$, we have at once $C = \frac{I}{E\omega}$, in which I and E are either maximum or mean square values of current and electromotive force respectively, and ω is 2π times the frequency. The current was measured by means of a Thomson milli-ampere balance, and difference of potential by a Thomson multicellular static voltmeter. The frequency may be easily found when the number of poles on the dynamo and the speed of the armature are known.

The data and results obtained by this method for one of the six condensers are given in Table I., and, for comparison, the measurements made by Thomson's method are given in Table II. In almost all cases the values of the capacity obtained by the method of mixtures were slightly higher than those found by the alternating current method. The equation $C = \frac{I}{E\omega}$ is true only when the impressed electromotive force is a sine-function of the time, and R is zero. Since neither of these conditions was strictly fulfilled, the results obtained were considered quite concordant, inasmuch as, taking all the measurements into consideration, the greatest difference between a single observation by the alternating current method and the method of mixtures was 7 per cent, while the greatest difference in the mean values obtained from a series of observations was 3.24 per cent.

TABLE I.

CAPACITY OF CONDENSER.

Volts. E.	Amperes. I.	ω	$C = \frac{I}{E\omega}$.
208.	.340	1127.	1.45
304.	.522	1127.	1.52
324.	.542	1110.	1.51
332.	.548	1110.	1.49
354.	.584	1097.	1.50
374.	.622	1102.	1.51
396.	.660	1108.	1.50
446.	.742	1108.	1.50
460.	.762	1104.	1.50
		Average . .	. . 1.50 (nearly)

TABLE II.

CAPACITY BY THOMSON'S METHOD.

Standard Capacity.	R_1.	R_2.	Capacity of Condenser.
.6 microfarads.	7000. ohms.	2752. ohms.	1.526 microfarads.
.4 "	7000. "	1835. "	1.526 "
.2 "	8000. "	1050. "	1.524 "
.1 "	8000. "	524. "	1.526 "
		Average . .	. 1.525 "

These results indicate that, for practical purposes, this method may be used where apparatus for more refined measurements is not available.

Since it was not known to what extent the capacity was dependent upon the temperature, a few determinations were made for the purpose of investigating this point. One of the con-

densers was placed in a tank of water, heated by means of a Bunsen's burner beneath, and at intervals the source of heat was removed and the capacity was measured by Thomson's method of mixtures, the temperatures of the water being noted each time. The variation of the capacity for rising and falling temperatures is graphically shown in Fig. 1. The capacity changes

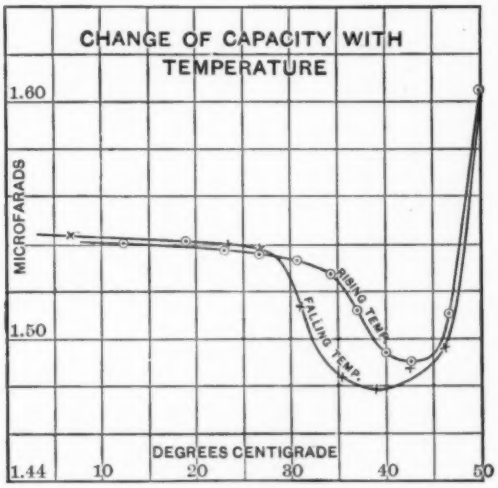


Fig. 1.

very little between 0° and 30° C. Above 30° the capacity decreases until a temperature of 40° is reached, while beyond this the capacity rises quite rapidly with increase of temperature. The reading for capacity was not taken until it became constant, at which time the temperatures of the condenser and of the water were assumed to be the same. It is not easy to explain the causes which give this curve its form.

In order to determine the leakage through the dielectric, one of the condensers was given a charge, and then allowed to stand several hours, with its terminals connected to the multicellular voltmeter. The falling off in the potential is shown by curve I., Fig. 2. For comparison, the theoretical exponential curve, showing the leakage through a dielectric resistance of 25,000 megohms, is plotted, curve II., in the same figure. Several values of the dielectric resistance of the condenser were calculated from curve

I., on the assumption that, when t is small, the expression $R = \frac{t}{C \log_e \frac{V_0}{V_t}}$ is approximately correct. These values are plotted

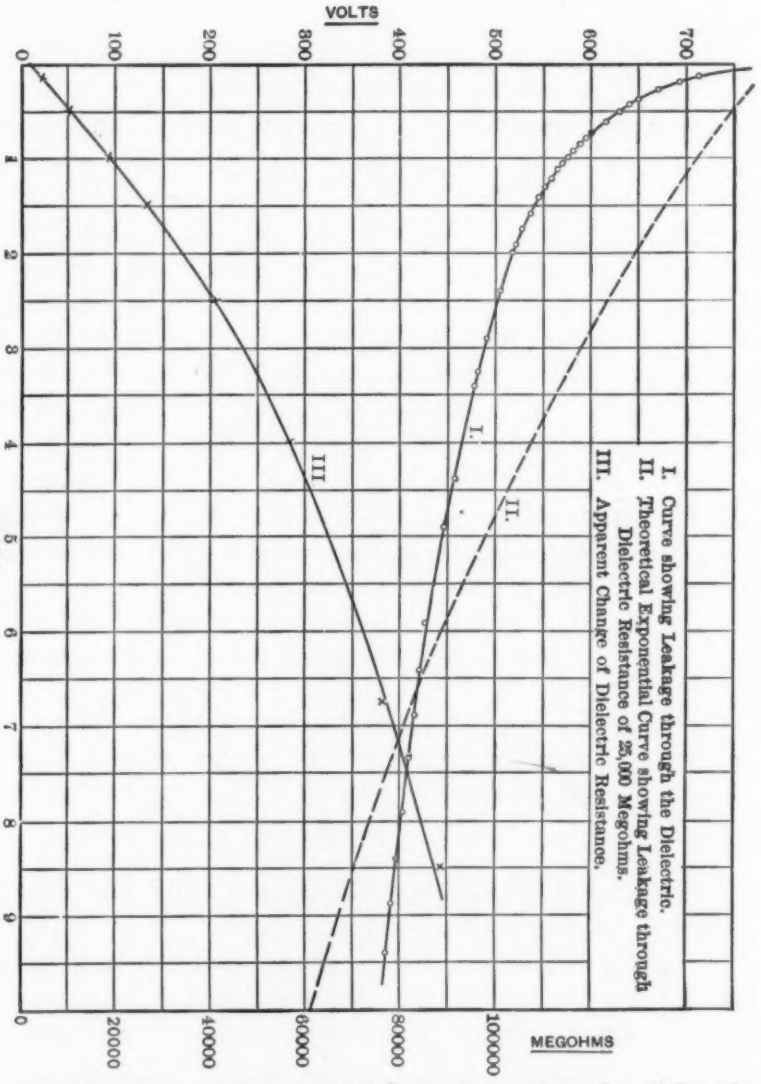


Fig. 2.

in curve III., Fig. 2, from which it appears that the dielectric resistance increases very greatly with increase of time and decrease of potential. This apparent increase is probably due to

the soaking in of the charge, which would cause a more rapid falling off in the potential at first than the falling off due to loss of charge through leakage.

Since some idea of the loss that occurs in a condenser when rapidly charged and discharged may be had from the amount of heat liberated, one of the condensers was subjected for five hours to an alternating electromotive force of 500 volts, at which potential the current is about three-fourths of an ampere, when the frequency is 160 alternations per second. The temperature of the condenser, which was well covered with cotton batting to prevent radiation, was read at intervals of ten minutes. Curves were plotted showing the change of temperature of the room as well as that of the condenser. It was ascertained that under the above conditions, if there were no radiation, the condenser would heat at the rate of very little over 1° C. an hour. This amount of heating shows the loss to be small, and therefore, if there is hysteresis in the dielectric, it must be small.

The power expended in a condenser may be ascertained by the three-voltmeter method. In a perfect condenser, the current is 90° ahead of the impressed electromotive force, this indicating that no power is lost, since the current has no component in the direction of the electromotive force. Any change in the angle of phase from 90° would cause the current to have a component in

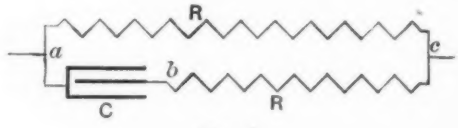


Fig. 3.

the direction of the electromotive force, which would show that there was some expenditure of energy.

A circuit, as shown in Fig. 3, was subjected to an alternating electromotive force and the electromotive force was measured between *a* and *b*, *a* and *c*, and *b* and *c*. The values obtained are given in Table III.

TABLE III.

E. M. F. between $a-b$.	E. M. F. between $a-c$.	E. M. F. between $b-c$.
1165.	568.	1009.
1151.	564.	1003.
1178.	590.	1029.
Average, 1165.	Average, 574.	Average, 1014.

Using the averages of the three values of the electromotive force obtained for each, the electromotive force triangle shown in Fig. 4 was drawn. In this triangle the electromotive force *ab* impressed upon the condenser is so nearly at right angles to the current, which is in the direction of *cb*, that the loss is again shown to be very small.

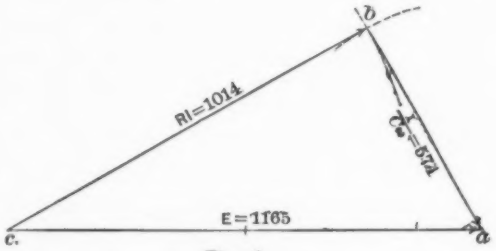


Fig. 4.

An attempt was made to obtain an hysteresis loop for a condenser by plotting a curve showing the relation between the charge and increasing and decreasing potentials. The condenser was charged up to any desired potential, measured by a multicellular voltmeter, and was then discharged through a ballistic galvanometer to find the quantity of charge. By successively recharging to different potentials, and discharging through the galvanometer, the corresponding quantities of charge were found. To show the relation between the potential and the quantity of charge, a curve was plotted in which abscissæ represented potentials, and ordinates represented the quantities of charge for increasing values of potential. To obtain the corresponding curve for decreasing potentials, the condenser was charged to a certain high potential, and then allowed to discharge through a high resistance to any desired potential, the quantity of charge at this potential being determined as

before by discharge through the galvanometer. This operation was repeated, the condenser being charged to the same high potential in each case, and allowed to discharge to some lower potential. If the curves drawn for increasing and decreasing potentials enclosed an area, it would indicate an hysteresis loss; but the two curves so nearly coincided that no loop was discernible. As the results of later experiments show the presence of hysteresis, it is probable that the time element had some effect. The necessary delay in measuring the potential doubtless permitted the condenser to acquire the same charge irrespective of whether the potential was raised or lowered to the particular final value.

Since considerable may be learned about the behavior of a condenser subjected to an alternating electromotive force from the instantaneous curves for electromotive force and current, it was thought that the subject of hysteresis in the dielectric might well be investigated in this way. From such curves the power expended at any instant may be found by multiplying the instantaneous value of the electromotive force by the corresponding value of the current. By obtaining in this way a series of values of the power, a watt curve may be plotted showing the amount of energy supplied to the condenser and the amount taken from it. From this curve the power wasted and the efficiency of the condenser may be determined. When the current and time of flow are known, the quantity of charge in the condenser at any time may be found according to the relation $q=\int idt$. The quantity of charge at any time during a cycle, and also the potential being known, a curve may be drawn showing graphically the relation between the two.

The arrangement of the apparatus for finding instantaneous values of the impressed electromotive force, of the electromotive force at the terminals of the condenser, and the electromotive force at the terminals of a non-inductive resistance in series with it, is shown in Fig. 5. The alternator is represented at A; a row of twenty-two incandescent lamps in series, by means of which the impressed electromotive force could be measured in two steps, is shown by R_1; the condenser to be tested is C_1, while R_2 is a row

of twenty-five incandescent lamps in series with the condenser. The several electromotive forces were measured with the multicellular voltmeter represented by V, to which the condenser C_2 was connected in multiple so as to prevent errors due to leakage.

The electromotive force at any particular part of the cycle was obtained by means of the contact-maker M, through which the voltmeter circuit was completed for an instant once during every revolution of the armature. The contact can be so adjusted as to complete the circuit at any part of the revolution as desired. For convenience in making connections, the points between which it was desired to measure the difference of potential were joined to suitable terminals G_1, G_2, G_3 on a switch-board. Readings were taken at intervals of $2\frac{1}{2}°$ on the contact-maker. The alternator having ten poles, there are five complete periods for every revolution of the armature, hence $72°$ on the contact-maker corresponds to a cycle. From the data thus obtained, the instantaneous electromotive force curves shown in Fig. 6 are plotted.

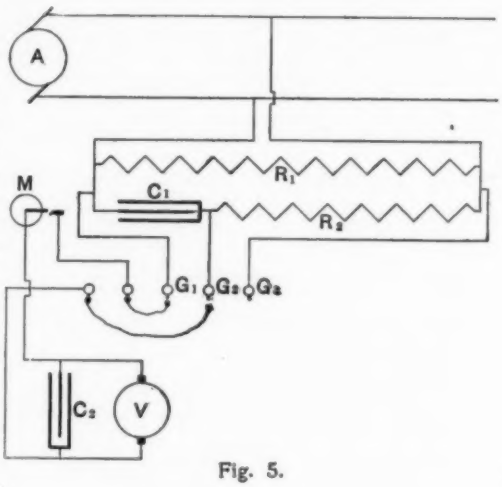


Fig. 5.

The current in the condenser circuit was found by calculating the current in the non-inductive lamp-resistance, R_2, according to Ohm's law. The resistance R_2 was previously determined by the fall of potential method. Since the condenser current is in phase with the electromotive force of curve II., Fig. 6, and has its ordinates proportional to those of curve II., this may also be taken to represent the instantaneous values of the current when a suitable scale is applied. Multiplying corresponding ordinates of curves II. and III., the watt curve given in Fig. 7 was obtained. From the way in which this curve was plotted, the areas enclosed above the

X-axis represent energy imparted to the condenser, while areas below represent the energy given out by it. Taking the difference between the upper and lower areas, the power lost was

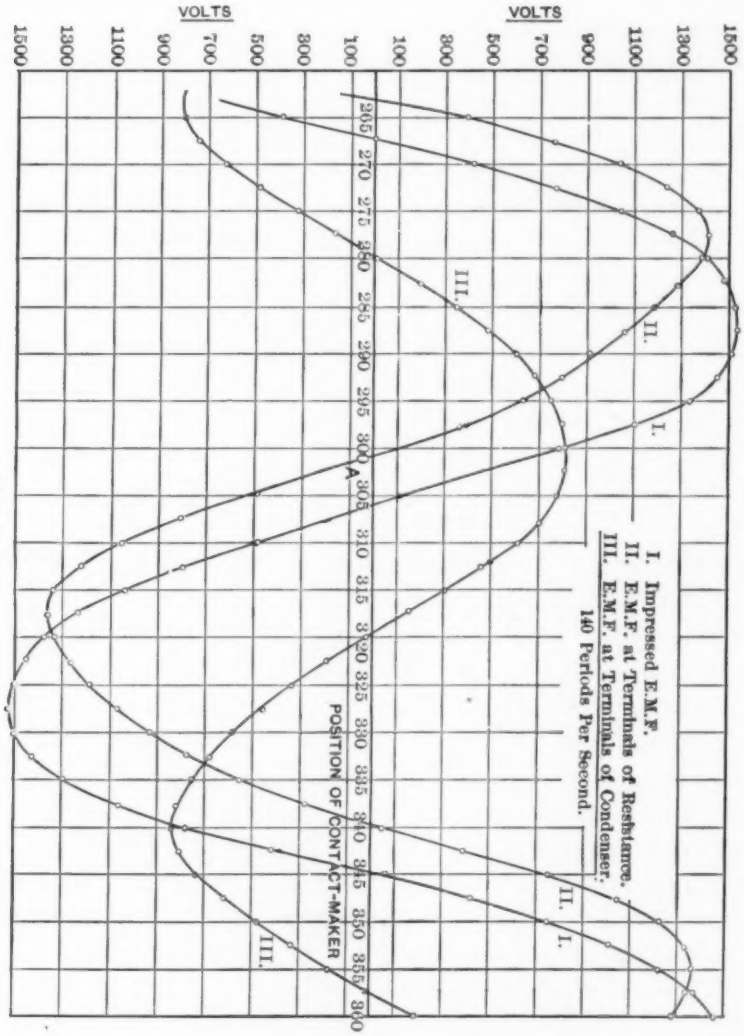


Fig. 6.

calculated to be 4.4 watts, which makes the efficiency of the condenser 96.9 per cent. In a similar preliminary run, the loss was found to be 2.47 watts, corresponding to an efficiency of 98.05 per cent.

The method devised for obtaining an hysteresis loop from the instantaneous curves for current and electromotive force is as follows. Since the quantity of electricity that flows through a circuit

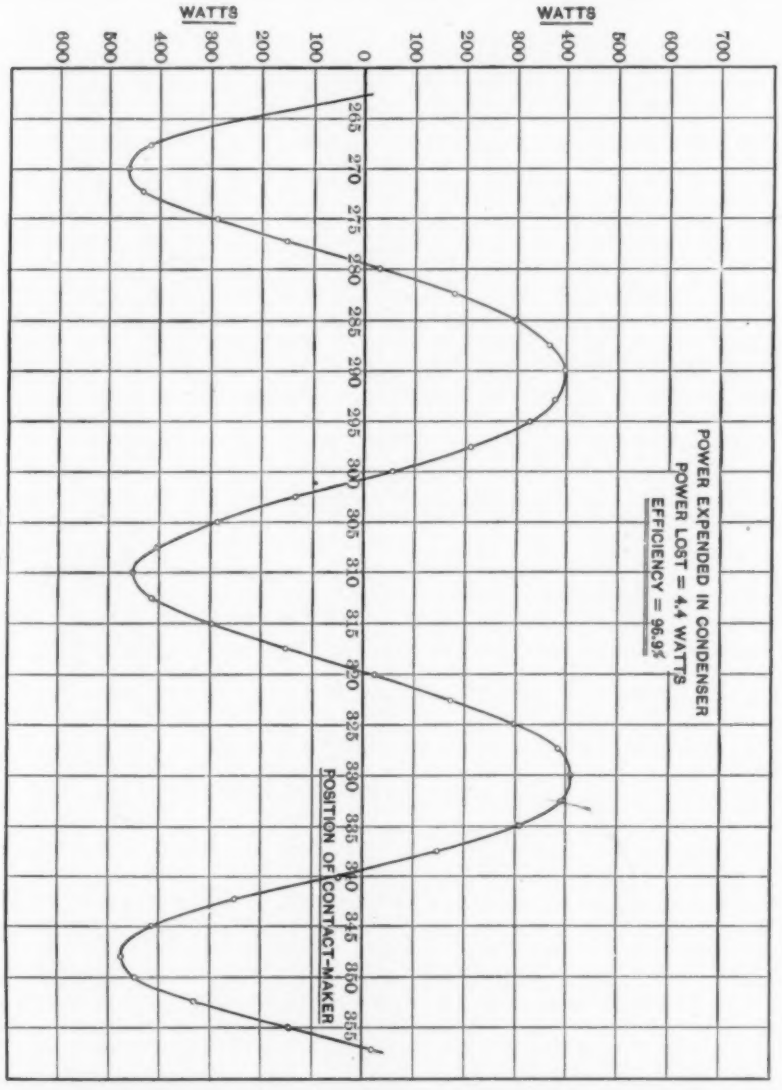


Fig. 7.

in the time dt is idt, if we start with no charge in a condenser and measure the value of the current from instant to instant, the total charge in the condenser at any time may be found by summing up

the instantaneous increments of charge. The total quantity of charge at the time t is $q=\int_0^t idt$, and is represented by the area enclosed between the X-axis, the current curve, and ordinates

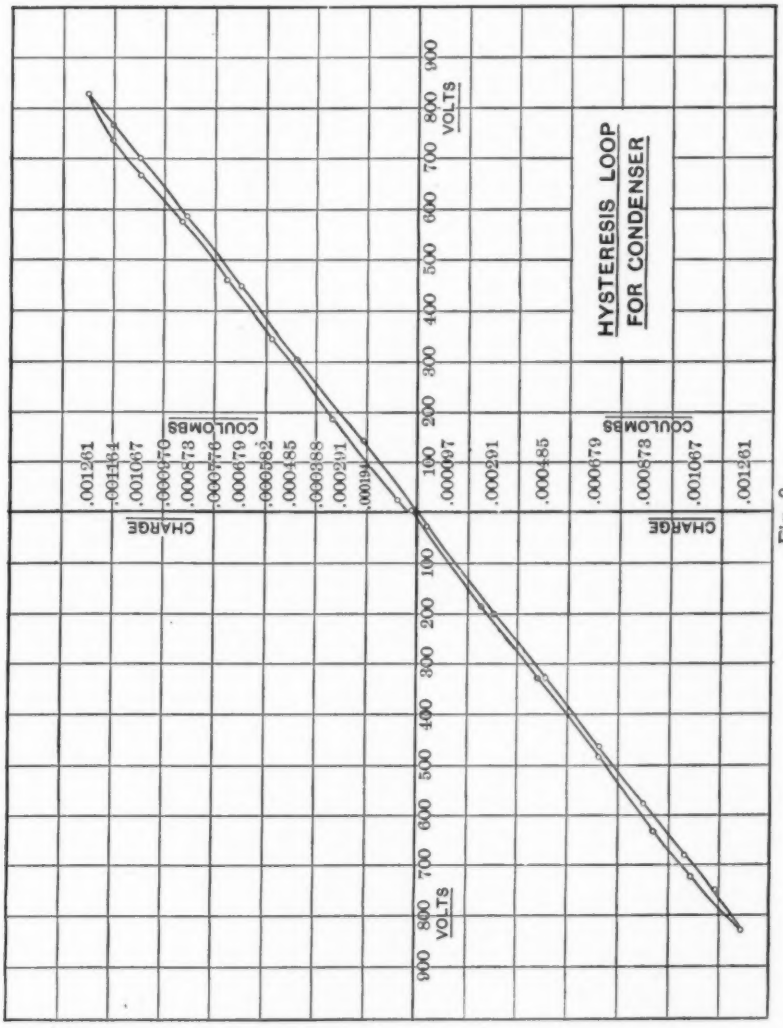


Fig. 8.

drawn at the points where $t=0$ and $t=t$. It being impossible to tell from the current curve at what point of the cycle the condenser contained no charge, the point A, Fig. 6, where the current is zero and the charge a maximum, was taken as a convenient place

to start in obtaining the quantities of charge at different parts of the cycle. Between two successive points where the current is zero, the condenser becomes discharged, and then charged in the opposite sense, so that the area of one of the loops, enclosed between the current curve and the X-axis, represents double the maximum charge. Starting from A and working toward the left, the quantity of charge in the condenser was found, for different potentials, by subtracting successive areas. Similarly the quantities of charge at different parts of the cycle to the right of A were found by adding, algebraically, successive areas, remembering that areas below the X-axis are negative. The loop shown in Fig. 8 was obtained by plotting potentials as abscissæ and quantities of charge, obtained in the way described, as ordinates. A similar loop was obtained from the data of the preliminary run. Now the area of this loop is a measure of the amount of energy dissipated in the condenser per cycle, being equal to $\int vdq$. The area, measured by an Amsler planimeter, was found to be equivalent to $.0512 \times 10^7$ ergs. As there were 140 complete alternations per second, the rate of dissipation of energy in the condenser is $.0512 \times 10^7 \times 140 = 7.17 \times 10^7$ ergs per second $= 7.17$ watts. This result may be considered a rough check on the previous work, but cannot be depended upon by itself, inasmuch as the accurate determination of such a long and thin area is not readily made. The nature of the result is interesting, although of little quantitative value.

In alternating-current circuits the neutralization of the effects of self-induction by capacity opens a large field for work. How far this neutralization can be effected depends upon the nature of the impressed electromotive force and the condenser itself. With a perfect condenser and an harmonic electromotive force it would be possible to completely annul the effect of self-induction. With an alternating current which is not harmonic and a condenser which dissipates a certain amount of energy, *i.e.* is not perfect, complete neutralization of self-induction can only be approximated to. Experiments were made with the condensers in question, and curves obtained showing the variation in the current for changes in capacity for certain circuits. These

curves showed a maximum point for the critical value of the capacity, but this was not as high or as well marked as the theoretical curves[1] plotted on the supposition that the condenser was perfect and the electromotive force harmonic.

In this paper we have given the results of an investigation of the loss in the dielectric of a condenser for one particular frequency. A complete investigation of the subject of dielectric hysteresis would include the determination of this loss for different frequencies and potentials, and also for different dielectrics. The law for dielectric hysteresis could thus be ascertained. The purpose of this paper has been merely to call attention to the existence of hysteresis and a method for its determination.

PHYSICAL LABORATORY OF CORNELL UNIVERSITY,
June 20, 1893.

[1] Alternating Currents, by Bedell and Crehore, p. 139.

ON THE INFRA-RED SPECTRA OF THE ALKALIES.[1]

II.

By Benjamin W. Snow.

V. THE ENERGY SPECTRA OF THE ALKALIES.

THE problem which had to be solved when the various alkalies were burned in the electric arc consisted in finding a means whereby the metals or their salts could be continuously and uniformly introduced. The following methods, among others, appeared the most feasible: — 1. To surround the lower (positive) carbon with a crucible in which could be placed the salt to be investigated. 2. To throw, with the aid of a suitable apparatus, a jet of the salt solution in the form of a fine spray directly into the arc. 3. To bore out the carbon and to fill with the salt to be burned the cavity thus formed, as the wick is imbedded in a candle. 4. To mix with the powdered carbon, during the process of manufacture, a sufficient quantity of the metallic salt, and to make from this mixture the special carbons to be used. The first three of these methods were tried, the third giving the best results, so that this was the means which was uniformly used in the actual experiments.

In the axis of the positive (lower) carbon, 8 mm. in diameter, was bored a hole 3 mm. in diameter, in which the dry and finely powdered salt was pounded with a hammer to a solid core. The negative carbon was also prepared in a similar manner, with the exception that the diameter of the cavity was only 1.5 mm.

As soon as the electric lamp was set in operation, the glowing of the carbon points, in which the salt was imbedded, caused the

[1] Continued from page 50.

salt to boil, and to send a perfectly uniform stream of metallic vapor directly into the arc.

The appearance of the arc was completely changed by this introduction of the salt. Instead of the peculiar bluish violet color, was seen at once the characteristic color of the metallic vapor. Even the form changed immediately, from the little spherical ball playing uniformly about the two carbon ends, into a slender ellipsoidal figure whose vertices were never at rest. These extremities in general rotated slowly around the two rings formed by the ends of the cored carbons (see Fig. 8). Since it was found impossible to obviate this wandering of the arc, an assistant was secured who, throughout all the following investigations, changed the position of the projecting lens, as the arc moved, so that the slit of the spectrometer, upon which a real image of the arc was cast, was always symmetrically and uniformly illuminated. The length of the arc was, moreover, examined before each observation and regulated when necessary.

Fig. 8.

The consumption of electrical energy in the lamp was also quite different when carbons were used which were prepared in the manner already described. It was found by trial that to maintain the current constant at 7.7 ampères, a resistance of six ohms had to be included in the lamp circuit when the solid carbons were used, but that this resistance must be increased to ten ohms when carbons were used containing the core of salt. In both cases the length of the arc remained the same. Whether this change was due to a decreased resistance of the arc when filled with metallic vapor, or to a smaller counter-electromotive force of the arc, could not be decided without further experiments. Since this phenomenon had no vital connection with the more immediate objects of the investigation, no further experiments were made bearing upon this point.

Inasmuch as the employment of the different salts of a metal gave similar results, as, for instance, metallic sodium, sodium carbonate, and sodium chloride, only the chlorides of the metals were used in the investigation of the other alkalies.

With one exception, the salts employed were obtained in the

purest possible condition from the Trommsdorff Chemical Works in Erfurt. Only in the investigation of the chloride of sodium was the ordinary commercial article used. Below is found the number of definitive experiments carried through with each salt :—

With NaCl, 6;
" KCl, 4;
" LiCl, 3;
" CsCl, 3;
" RbCl, 2.

So complete was the uniformity of the results obtained with each metal that no hesitation is felt in selecting at random, from the material at hand, a series of observations illustrative of each group.

The contents of the following tables may be found in graphical form in Figs. 5–9, Plates II. and III. Wave-lengths are again chosen as abscissæ, and galvanometer deflections as ordinates. Even these curves can in no way lay claim to representing the true distribution of energy in the normal spectrum, since here also the lines partially, at least, overlap, as may be clearly observed in the curve for sodium from the fact that the yellow D-line does not appear double.

TABLE V.

SODIUM. $k = \frac{1}{135000}$° C.

ϕ	λ	I	ϕ	λ	I
52° 37½′	0.3836 μ	0	48° 21½′	0.610 μ	29
28½′	0.3852 "	1	19′	0.616 "	91
23′	0.3870 "	0	16′	0.622 "	11
20½′	0.3880 "	0	10½′	0.637 "	13
13′	0.3908 "	0	7½′	0.644 "	22
7′	0.3932 "	31	6′	0.648 "	13
3½′	0.3950 "	2	3½′	0.654 "	8
			½′	0.663 "	6
51° 59½′	0.3967 "	31			
55½′	0.3983 "	0	47° 57½′	0.671 "	26
11½′	0.4218 "	1	56′	0.676 "	10
8½′	0.4236 "	42	54½′	0.680 "	10
5½′	0.4255 "	4	53′	0.685 "	4
			51½′	0.690 "	7
50° 10½′	0.4650 "	0	50′	0.695 "	5
7′	0.4677 "	11	48½′	0.699 "	7
4′	0.4703 "	1	47′	0.704 "	6
			45½′	0.710 "	14
49° 37′	0.4963 "	6	44′	0.714 "	13
34′	0.4996 "	62	42½′	0.720 "	12
31′	0.5028 "	4	41′	0.726 "	9
22′	0.5130 "	7	39½′	0.731 "	9
19′	0.5164 "	12	38′	0.737 "	10
16′	0.5200 "	3	36½′	0.743 "	5
13′	0.5240 "	4	35′	0.749 "	6
10½′	0.5271 "	16	33½′	0.755 "	9
7½′	0.5310 "	1	32′	0.761 "	12
			30½′	0.768 "	24
48° 50½′	0.5560 "	10	29′	0.775 "	12
48′	0.5600 "	19	27½′	0.781 "	6
46½′	0.5625 "	15	25½′	0.789 "	8
43′	0.5685 "	186	24′	0.796 "	8
39½′	0.5742 "	21	22½′	0.803 "	22
38′	0.5772 "	20	21′	0.811 "	166
36½′	0.5800 "	27	20½′	0.814 "	603
31½′	0.5892 "	877	19½′	0.818 "	659
28½′	0.5950 "	96	18′	0.825 "	170
24½′	0.6040 "	28	16½′	0.833 "	27

TABLE V. — *Continued.*

SODIUM. $k = \frac{1}{135000}$° C.

ϕ	λ	I	ϕ	λ	I
47° 15′	0.841 μ	22	46° 21½′	1.317 μ	5
13½′	0.849 "	18	20′	1.337 "	4
12′	0.857 "	18	18½′	1.358 "	3
10½′	0.865 "	11	17′	1.380 "	5
9′	0.874 "	8	15½′	1.399 "	4
7½′	0.884 "	6	14′	1.421 "	3
6′	0.895 "	6	12½′	1.442 "	3
4½′	0.905 "	6	11′	1.461 "	4
3′	0.916 "	4	9½′	1.482 "	3
1½′	0.926 "	7	8′	1.503 "	2
0′	0.938 "	7	6½′	1.523 "	3
			5′	1.543 "	2
46° 58½′	0.949 "	5	3½′	1.563 "	0
57′	0.962 "	6	2′	1.583 "	6
55′	0.977 "	5	½′	1.603 "	2
53½′	0.988 "	8			
52′	1.003 "	8	45° 59′	1.624 "	3
50½′	1.014 "	4	57½′	1.645 "	2
49′	1.028 "	6	56′	1.667 "	0
47½′	1.041 "	5	54′	1.694 "	4
46′	1.055 "	7	52½′	1.713 "	6
44½′	1.067 "	12	51′	1.733 "	5
43′	1.082 "	12	49½′	1.755 "	17
41½′	1.096 "	11	48′	1.776 "	12
40′	1.111 "	45	46½′	1.795 "	17
38½′	1.127 "	415	45′	1.816 "	11
37′	1.143 "	112	43½′	1.836 "	33
35½′	1.158 "	24	42′	1.857 "	25
34′	1.176 "	11	40½′	1.878 "	11
32½′	1.191 "	7	39′	1.898 "	5
31′	1.208 "	8	37½′	1.918 "	4
29½′	1.223 "	10	36′	1.938 "	2
28′	1.242 "	30	34½′	1.958 "	0
26½′	1.257 "	16	33′	1.978 "	0
25′	1.276 "	8	31½′	2.000 "	0
23′	1.300 "	6			

TABLE VI.

POTASSIUM. $k = \frac{1}{127000}$° C.

ϕ	λ	I	ϕ	λ	I
52° 35½′	0.3830 μ	0	47° 54½′	0.680 μ	8
29½′	0.3850 "	2	51′	0.691 "	74
23½′	0.3870 "	0	49′	0.698 "	13
21′	0.3877 "	0	44½′	0.713 "	7
14′	0.3903 "	0	39′	0.733 "	12
			36′	0.745 "	27
51° 46′	0.4030 "	0	31′	0.767 "	1443
43′	0.4045 "	30	30′	0.770 "	622
40′	0.4060 "	0	28½′	0.777 "	82
12′	0.4214 "	2	27′	0.783 "	31
9′	0.4233 "	16	25½′	0.790 "	31
6′	0.4251 "	0	24′	0.796 "	27
			22½′	0.803 "	36
50° 17½′	0.4590 "	2	21′	0.811 "	34
			19½′	0.818 "	22
49° 46′	0.4870 "	2	18′	0.825 "	16
41½′	0.4915 "	0	16½′	0.832 "	16
39½′	0.4942 "	3	15′	0.841 "	17
32′	0.5023 "	0	13½′	0.850 "	14
25′	0.5095 "	4	12′	0.857 "	12
23½′	0.5113 "	5	10½′	0.866 "	12
20½′	0.5148 "	0	9′	0.876 "	13
8′	0.5307 "	0	7½′	0.886 "	12
5½′	0.5340 "	9	6′	0.896 "	9
4′	0.5362 "	6	4½′	0.906 "	8
2′	0.5388 "	0	3′	0.917 "	10
			1′	0.931 "	7
48° 40′	0.5737 "	1			
36½′	0.5800 "	14	46° 59½′	0.943 "	20
34′	0.5848 "	7	58′	0.954 "	20
31½′	0.5892 "	55	56½′	0.966 "	9
28½′	0.5950 "	3	55′	0.977 "	8
21½′	0.6093 "	2	53½′	0.990 "	7
19′	0.616 "	6	52′	1.002 "	9
10′	0.638 "	8	50½′	1.015 "	8
8′	0.643 "	11	49′	1.029 "	14
5½′	0.649 "	8	47½′	1.042 "	16

TABLE VI. — *Continued.*

POTASSIUM. $k = \frac{1}{127000}$° C.

ϕ	λ	I	ϕ	λ	I
46° 46′	1.055 μ	21	46° 3½′	1.563 μ	6
44½′	1.068 "	17	2′	1.583 "	3
43′	1.082 "	102	½′	1.603 "	2
42½′	1.090 "	57			
41½′	1.096 "	25	45° 59′	1.625 "	4
40′	1.110 "	13	57′	1.652 "	2
38½′	1.127 "	35	55½′	1.672 "	2
37′	1.144 "	207	54′	1.693 "	4
35½′	1.158 "	389	52½′	1.713 "	3
34′	1.175 "	34	51′	1.733 "	3
32½′	1.193 "	14	49½′	1.755 "	2
31′	1.209 "	126	48′	1.776 "	5
29½′	1.225 "	190	46½′	1.795 "	0
28′	1.243 "	43	45′	1.816 "	0
26′	1.264 "	10	43½′	1.836 "	2
24½′	1.283 "	11	42′	1.857 "	5
23′	1.301 "	10	40½′	1.876 "	3
21½′	1.317 "	8	39′	1.898 "	2
20′	1.336 "	10	37½′	1.918 "	4
18½′	1.359 "	10	36′	1.938 "	0
17′	1.379 "	5	34½′	1.958 "	0
15½′	1.400 "	5	33′	1.978 "	0
14′	1.420 "	8	31½′	2.000 "	0
12½′	1.440 "	19	30′	2.021 "	0
11′	1.461 "	60	28½′	2.042 "	0
9½′	1.482 "	56	27′	2.062 "	0
8′	1.503 "	49	25′	2.087 "	0
6½′	1.523 "	12	23½′	2.108 "	0
5′	1.543 "	5	22′	2.128 "	0

TABLE VII.

LITHIUM. $k = \frac{1}{134000}$° C.

ϕ	λ	I	ϕ	λ	I
52° 33½′	0.3838 μ	2	47° 58′	0.670 μ	1191
26′	0.3860 "	2	56′	0.676 "	57
22′	0.3875 "	2	54½′	0.680 "	29
16′	0.3900 "	0	53′	0.684 "	24
11½′	0.3913 "	8	51½′	0.689 "	18
4′	0.3947 "	0	50′	0.694 "	15
			48½′	0.699 "	11
51° 29½′	0.4116 "	1	47′	0.704 "	10
25½	0.4140 "	58	45½′	0.709 "	10
22½′	0.4155 "	5	44′	0.715 "	11
11′	0.4220 "	0	42½′	0.720 "	8
8½′	0.4238 "	11	41′	0.725 "	7
4½′	0.4260 "	0	39½′	0.732 "	6
½′	0.4288 "	10	38′	0.737 "	5
			36½′	0.743 "	10
50° 57½′	0.4305 "	2	35′	0.749 "	6
18′	0.4590 "	8	33½′	0.754 "	6
14½′	0.4615 "	331	32′	0.760 "	11
10½′	0.4650 "	7	30½′	0.768 "	12
			29′	0.775 "	8
49° 37½′	0.4958 "	2	27½′	0.781 "	9
34½′	0.4990 "	34	26′	0.787 "	17
31½′	0.5023 "	2	24½′	0.794 "	18
			22½′	0.803 "	151
48° 59½′	0.5420 "	8	21¾′	0.807 "	292
34½′	0.5840 "	8	21′	0.811 "	238
31½′	0.5892 "	51	19½′	0.819 "	22
28½′	0.5950 "	8	18′	0.825 "	15
26½′	0.5993 "	5	16½′	0.833 "	12
25′	0.6025 "	20	15′	0.841 "	11
21½′	0.6102 "	570	13½′	0.849 "	15
18′	0.6180 "	17	12′	0.857 "	12
12½′	0.6308 "	7	10½′	0.865 "	10
5½′	0.6490 "	18	9′	0.876 "	9
½′	0.663 "	46	7½′	0.885 "	8

TABLE VII. — *Continued.*

LITHIUM. $k = \frac{1}{134000}$° C.

ϕ	λ	I	ϕ	λ	I
47° 6′	0.895 μ	5	46° 15½′	1.397 μ	2
4½′	0.904 "	7	14′	1.420 "	3
3′	0.916 "	9	12½′	1.440 "	3
1½′	0.926 "	11	11′	1.462 "	2
0′	0.937 "	11	9½′	1.482 "	4
			8′	1.502 "	3
46° 58½′	0.950 "	9	6½′	1.522 "	2
57′	0.962 "	9	5′	1.543 "	3
55½′	0.973 "	4	3½′	1.563 "	2
54′	0.987 "	8	2′	1.583 "	1
52′	1.002 "	5	½′	1.603 "	4
50½′	1.014 "	5			
49′	1.028 "	6	45° 59′	1.625 "	2
47½′	1.041 "	6	57½′	1.645 "	2
46′	1.054 "	5	56′	1.665 "	0
44½′	1.067 "	7	54½′	1.685 "	0
43′	1.082 "	14	53′	1.706 "	9
41½′	1.096 "	8	51′	1.732 "	10
40′	1.111 "	9	49½′	1.755 "	12
38½′	1.127 "	9	48′	1.776 "	16
37′	1.144 "	6	46½′	1.796 "	13
35½′	1.157 "	7	45′	1.816 "	14
34′	1.176 "	7	43½′	1.835 "	11
32½′	1.192 "	9	42′	1.857 "	10
31′	1.208 "	7	40½′	1.876 "	8
29½′	1.225 "	8	39′	1.898 "	12
28′	1.242 "	5	37½′	1.918 "	8
26½′	1.257 "	7	36′	1.937 "	3
25′	1.276 "	8	34½′	1.958 "	0
23½′	1.294 "	4	33′	1.978 "	0
21½′	1.317 "	4	31½′	2.000 "	0
20′	1.336 "	3	30′	2.020 "	0
18½′	1.357 "	2	28½′	2.042 "	0
17′	1.378 "	3			

TABLE VIII.

RUBIDIUM. $k = \frac{1}{132000}$° C.

ϕ	λ	I	ϕ	λ	I
52° 30′	0.3832 μ	0	48° 14′	0.627 μ	23
24′	0.3868 "	0	11′	0.633 "	12
20½′	0.3880 "	0	6′	0.648 "	10
16½′	0.4190 "	0	1′	0.662 "	8
51° 14′	0.4200 "	6	47° 58½′	0.669 "	11
12′	0.4215 "	4	55½′	0.677 "	8
9½′	0.4230 "	8	49½′	0.696 "	5
6½′	0.4250 "	0	45′	0.711 "	8
			41′	0.726 "	21
50° 18′	0.4588 "	0	39½′	0.731 "	17
			38′	0.737 "	19
49° 19′	0.5163 "	3	35½′	0.747 "	15
15′	0.5215 "	5	33½′	0.755 "	31
12¾′	0.5242 "	2	31½′	0.763 "	128
10½′	0.5270 "	6	30½′	0.768 "	188
6′	0.5332 "	2	29′	0.775 "	414
3½′	0.5367 "	4	27½′	0.781 "	262
½′	0.5406 "	2	25½′	0.791 "	443
			24½′	0.794 "	279
48° 58½′	0.5435 "	4	23′	0.801 "	56
56′	0.5473 "	2	21½′	0.808 "	38
48½′	0.5592 "	9	20′	0.815 "	32
45½′	0.5642 "	3	18½′	0.823 "	42
43½′	0.5676 "	1	17′	0 831 "	24
41½′	0.5710 "	7	15½′	0.839 "	25
38½′	0.5762 "	2	14′	0.846 "	50
35½′	0.5820 "	3	12½′	0.854 "	14
31½′	0.5892 "	26	11′	0.863 "	11
28¾′	0.5942 "	5	9½′	0.872 "	45
25′	0.6020 "	4	8′	0.882 "	59
23′	0.607 "	8	6½′	0.892 "	17
21′	0.611 "	6	5′	0.902 "	10
19′	0.616 "	12	3½′	0.912 "	11
17′	0.620 "	11	2′	0.923 "	9
15½′	0.624 "	10	½′	0.934 "	9

TABLE VIII. — *Continued.*

RUBIDIUM. $k = \frac{1}{132000}$° C.

ϕ	λ	I	ϕ	λ	I
46° 59′	0.946 μ	10	46° 8½′	1.498 μ	64
57½′	0.957 "	7	7′	1.517 "	71
55½′	0.972 "	7	5½′	1.535 "	16
54′	0.986 "	21	4′	1.556 "	6
52½′	0.998 "	160	2½′	1.576 "	5
51′	1.011 "	32	1′	1.597 "	5
49½′	1.023 "	9			
48′	1.038 "	7	45° 59½′	1.618 "	5
46½′	1.050 "	8	57½′	1.645 "	4
45′	1.064 "	11	56′	1.666 "	4
43½′	1.077 "	10	54½′	1.685 "	2
42′	1.092 "	13	53′	1.705 "	3
40½′	1.105 "	11	51½′	1.725 "	3
39′	1.124 "	7	50′	1.747 "	3
37½′	1.138 "	13	48½′	1.770 "	0
36′	1.154 "	26	47′	1.790 "	0
34½′	1.170 "	10	45½′	1.812 "	0
33′	1.187 "	5	44′	1.831 "	3
31½′	1.203 "	6	42½′	1.850 "	1
30′	1.218 "	13	41′	1.870 "	0
28′	1.242 "	10	39½′	1.890 "	0
26½′	1.258 "	9	38′	1.913 "	0
25′	1.277 "	35	36½′	1.932 "	0
23½′	1.293 "	100	35′	1.952 "	0
22′	1.312 "	197	33½′	1.972 "	0
20½′	1.330 "	151	32′	1.993 "	0
19′	1.352 "	83	30½′	2.014 "	0
17½′	1.372 "	16	29′	2.036 "	0
16′	1.393 "	15	27′	2.062 "	0
14½′	1.413 "	40	25½′	2.082 "	0
13′	1.434 "	82	24′	2.102 "	0
11½′	1.454 "	94	22½′	2.120 "	0
10′	1.476 "	102			

TABLE IX.

CÆSIUM. $k = \frac{1}{132000}$° C.

ϕ	λ	I	ϕ	λ	I
52° 41½′	0.3812 μ	0	48° 7½′	0.644 μ	10
32′	0.3842 "	0	5′	0.650 "	9
25′	0.3868 "	0	2½′	0.658 "	15
22½′	0.3875 "	0	½′	0.663 "	19
51° 17′	0.4186 "	0	47° 58′	0.670 "	46
14½′	0.4200 "	4	56′	0.676 "	11
12½′	0.4212 "	2	54′	0.681 "	17
10½′	0.4225 "	7	52½′	0.687 "	26
7½′	0.4243 "	0	50′	0.694 "	63
			47½′	0.702 "	20
50° 23½′	0.4545 "	0	45′	0.711 "	11
21′	0.4565 "	15	42′	0.721 "	47
18½′	0.4584 "	2	39′	0.733 "	13
16½′	0.4600 "	6	34½′	0.751 "	12
13½′	0.4625 "	0	32½′	0.759 "	43
			31′	0.766 "	65
49° 41′	0.4920 "	4	29′	0.775 "	175
38½′	0.4950 "	4	27½′	0.781 "	95
0′	0.5412 "	4	26′	0.788 "	104
			24½′	0.794 "	105
48° 52½′	0.5528 "	7	23′	0.801 "	77
50′	0.5570 "	4	21½′	0.808 "	45
46′	0.5635 "	7	20′	0.815 "	29
43′	0.5686 "	2	18½′	0.823 "	26
38′	0.5772 "	4	17′	0.831 "	54
35′	0.5828 "	28	15½′	0.838 "	297
33½′	0.5856 "	23	14′	0.847 "	226
31½′	0.5892 "	54	12½′	0.854 "	59
28½′	0.5950 "	9	11′	0.863 "	151
25¾′	0.6010 "	14	9½′	0.872 "	140
23′	0.607 "	9	8′	0.882 "	345
20′	0.614 "	8	6½′	0.892 "	113
17½′	0.619 "	23	5′	0.902 "	154
15½′	0.624 "	8	3½′	0.913 "	83
13½′	0.629 "	12	2′	0.923 "	16
10½′	0.636 "	7	½′	0.934 "	9

TABLE IX. — *Continued.*

CÆLIUM. $k = \frac{1}{132000}$° C.

ϕ	λ	I	ϕ	λ	I
46° 59′	0.945 μ	10	46° 8½′	1.495 μ	19
57½′	0.956 "	9	7′	1.516 "	20
55½′	0.973 "	7	5½′	1.535 "	5
54′	0.986 "	129	4′	1.555 "	6
52½′	0.998 "	181	2½′	1.575 "	7
51′	1.011 "	17	1′	1.596 "	4
49½′	1.023 "	5			
48′	1.038 "	6	45° 59½′	1.617 "	3
46½′	1.050 "	4	58′	1.638 "	4
45′	1.064 "	6	56½′	1.658 "	3
43½′	1.077 "	6	55′	1.679 "	3
42′	1.092 "	4	53½′	1.700 "	2
40½′	1.105 "	6	51½′	1.725 "	2
39′	1.124 "	8	50′	1.748 "	2
37½′	1.138 "	8	48½′	1.768 "	1
36′	1.154 "	8	47′	1.789 "	2
34½′	1.170 "	4	45½′	1.807 "	1
33′	1.187 "	4	44′	1.830 "	0
31½′	1.202 "	7	42½′	1.850 "	0
30′	1.219 "	6	41′	1.871 "	0
28½′	1.236 "	4	39½′	1.890 "	0
27′	1.253 "	12	38′	1.912 "	0
25½′	1.269 "	13	36½′	1.930 "	0
23½′	1.293 "	27	35′	1.952 "	0
22′	1.312 "	64	33½′	1.972 "	0
20½′	1.327 "	80	32′	1.992 "	0
19′	1.352 "	31	30½′	2.012 "	0
17½′	1.372 "	7	29′	2.036 "	0
16′	1.392 "	11	27½′	2.055 "	0
14½′	1.412 "	35	26′	2.075 "	0
13′	1.433 "	38	24½′	2.095 "	0
11½′	1.453 "	51	22½′	2.120 "	0
10′	1.475 "	25	21′	2.142 "	0

VI. DISCUSSION OF THE OBSERVATIONS.

A hasty consideration of the curves of Plates II. and III. shows that the energy spectra of the alkalies have but little in common with that of the electric arc. While the latter produces, as has been shown above, the impression of a banded spectrum, the former is at once recognized as a spectrum composed of bright lines.

Kayser and Runge mention,[1] in their investigation, the fact that measurements upon the spectral lines of metals vaporized in the electric arc are often rendered difficult by the superposition of the metallic lines upon the banded carbon spectrum.

The present writer, however, has found, by following the method here described of filling the carbons with a core of the salt to be examined, that *the carbon spectrum is completely destroyed, and in its stead appears the pure metallic spectrum.* To attain this result, however, it is necessary to set the lamp in operation about five minutes before beginning the series of observations, since by the ignition of fresh carbons, the bands of the arc spectrum are often to be seen between the lines in the spectrum of the metal in question. After the lapse of this short interval of time, the bolometer, even when placed at the position of maximum heat in the arc spectrum (from $\lambda = .385\,\mu$ to $\lambda = .388\,\mu$), gives at most a deflection of but 2 or 3 mm., while large deflections at this same place are obtained when pure carbons are used.

It must be mentioned in this connection that to attain this result it is necessary that *both* carbons be filled with a core of salt, since when a solid cathode, for instance, is used, there always clings to the ends of the negative carbon a point of the characteristic violet light of the ordinary arc.

By increasing the dimensions of the cavity containing the salt in the carbons, the energy of the metallic lines may be many times increased. It was observed that a core 5 mm. in diameter increased the intensity of the lines nearly tenfold. On the other

[1] *Kayser* und *Runge*, Abh. d. K. Acad. d. Wiss. zu Berlin, 2. Abs. p. 3, 1889. Wied. Ann., **38**, p. 80, 1889.

hand, the use of such large quantities of salt was attended with other disadvantages which made it desirable to return to the original dimensions of the holes bored; viz. 3 mm. and $1\frac{1}{2}$ mm. respectively.

It is to be noticed in all these drawings that the lines become broader as the more distant portions of the infra-red are reached. This effect does not, however, correspond to a physical cause, but is due in part to the method of representation here chosen, in part also to the chromatic aberration of the lenses of the spectrometer. As the bolometer filament wanders through the entire spectrum, there is concentrated upon its surface a far greater number of rays of different wave-lengths in the infra-red than in the region visible to the eye. If the thread of the bolometer were to be drawn at different places in Figs. 4–9, Plates II. and III., its breadth at $\lambda = 2\,\mu$ would appear nearly forty times greater than at $\lambda = .4\,\mu$. The same is true of the image of the slit itself. Metallic lines which in the energy curve of the dispersion spectrum appear equally broad, must, accordingly, appear much broader at $\lambda = 2\,\mu$ than at $\lambda = .4\,\mu$, when wave-lengths are chosen as abscissæ instead of the angular deviation of the bolometer arm.

To this effect is added, as mentioned above, that due to the lack of achromatism of the lenses used. This failure is far more noticeable in the infra-red than in the visible portions of the spectrum, and as a result the lines become more indistinct the farther they lie in the direction of the long wave-lengths. Abney[1] has also remarked that the focus of a lens for infra-red rays is quite different from its focus for radiations of greater refrangibility.

The cause of this failing achromatism in a system of lenses is to be found in the different characters which the curves of dispersion for flint and crown glass show in the infra-red. Rubens, in his investigation "Ueber Dispersion Ultrarother Strahlen," has shown[2] that the curves of dispersion for specimens of flint glass become nearly straight in the infra-red, but yet are always curved in the same direction, while in the curves of dis-

[1] *Abney*, Phil. Trans., Pt. II., p. 658, 1880.
[2] *Rubens*, Wied. Ann., **45**, p. 255, 1892.

persion of crown glass a point of inflection occurs in the neighborhood of $\lambda = 1.4\,\mu$, which causes the direction of the curvature at this point to change.

The extent to which this effect was noticeable in the measurements of the energy spectra may be illustrated by an example. When the telescope was focused on a green potassium line, the bolometer detected between the limits $\lambda = 1.440\,\mu$ and $\lambda = 1.540\,\mu$ a broad, poorly defined band, which passed into an easily recognized, although not perfectly sharp line, as soon as the focus was adjusted upon the line potassium α.

In conducting the definitive measurements, the observing telescope was so adjusted that the line potassium α appeared perfectly sharp. It is believed by thus focusing upon this most distant visible line toward the infra-red end of the spectrum, that a portion at least of the defective achromatism was avoided.[1]

Before a short description of the several metallic spectra is given in detail, a word may be said regarding the influence upon the intensities of the lines of the absorption within the measuring apparatus. In the less refrangible regions of the visible spectrum and in those portions of the infra-red here investigated, the absorption is inconsiderable, as is proven in the paper by Rubens so frequently cited. As is there shown, strong absorption is not perceptible in the specimens of crown glass examined until a wave-length $\lambda = 2.3\,\mu$ is reached, while flint glass exhibits a far greater transparency for infra-red radiations. The absorption produced in the violet end of the spectrum by the lenses of the spectrometer and by the prism is, without doubt, of much greater magnitude, though the amount could hardly be estimated. In a recent paper, Professor Nichols and the present writer have shown[2] that even a single crown glass lens can produce heavy absorption in this portion of the spectrum. It may then be supposed that the violet bands in the spectrum of the electric arc are far more intense than they appear from the curves shown in Figs. 3 and 4, Plate I.

[1] It need hardly be mentioned here that these errors would not affect the position or wave-lengths of the lines, but would simply tend to diminish the intensity.

[2] *Nichols* and *Snow*, Phil. Mag. (5), **33**, p. 380, 1892.

VII. ON THE METALLIC LINES.

The visible region of the sodium spectrum contains a series of lines which may be readily observed with the eye, but which are so lacking in energy that they exert no appreciable effect upon the bolometer. On the other hand, the two violet lines at $\lambda = .3932\,\mu$ and $\lambda = .3967\,\mu$, coinciding, as they do, very closely with the Fraunhofer lines H and K of the solar spectrum, appear sharply defined in the drawing.[1] The green line at $\lambda = .5685\,\mu$, the yellow at $\lambda = .5892\,\mu$, D, and the red at $\lambda = .616\,\mu$ are the strongest in the visible region. In the infra-red are found, besides a series of weaker lines at $\lambda = .770\,\mu$, $\lambda = .855\,\mu$, $\lambda = .930\,\mu$, $\lambda = 995\,\mu$, $\lambda = 1.075\,\mu$, $\lambda = 1.245\,\mu$, two lines of unusual intensity at $\lambda = .818\,\mu$ and $\lambda = 1.132\,\mu$. The energy of the first of these two is approximately equal to that of the D-line, and, indeed, exceeds the latter in several series of observations. The bolometer shows, further, in the region between $\lambda = 1.70\,\mu$ and $\lambda = 1.90\,\mu$, the presence of a small amount of energy, which, if the proper focus could be adjusted, might be resolved into a line or a group of lines. It has, however, not been possible thus to analyse this diffuse radiation, which also appears in a similar position in the lithium spectrum.

Much simpler than the spectrum of sodium is that of lithium, which, besides this feeble amount of energy between $\lambda = 1.70\,\mu$ and $\lambda = 1.90\,\mu$, shows only a single line at $\lambda = .811\,\mu$ in the infra-red. There is here noticeable, in addition to the eight visible lines measurable with the bolometer, also the yellow sodium line. This metal occurred in small quantities as an impurity in all the salts investigated, and in all cases made its presence known by a more or less prominent D-line.

In the potassium spectrum, the largest portion of the energy is concentrated in the one line potassium α. While this line exerts only a feeble effect upon the eye, it is, as regards its radiant energy, by far the strongest line found in the spectra of the five alkalies investigated. This is then an illustration of the

[1] Professor Kayser has kindly pointed out that these two lines are undoubtedly due to the presence of a trace of calcium in the salt used.

fact that the human eye is far from sensitive to radiations of this wave-length.

A noticeable difference between the spectrum of this metal and that of sodium and lithium is the relatively luminous background upon which the lines are superposed. Infra-red lines were found at $\lambda = .840\,\mu$, $\lambda = .885\,\mu$, $\lambda = .950\,\mu$, $\lambda = 1.086\,\mu$, $\lambda = 1.155\,\mu$, and $\lambda = 1.220\,\mu$, besides the diffuse line mentioned above at $\lambda = 1.470\,\mu$. The other lines occurring at $\lambda = .589\,\mu$ and $\lambda = .815\,\mu$ in Fig. 6, Plate II., belong to sodium.

The spectra of rubidium and cæsium are richer in infra-red lines than those thus far considered. In the visible region the background upon which the lines of these metals seem to be projected appears more luminous than that of potassium — a fact which is undoubtedly closely connected with the whitish appearance which the arc assumes as soon as these metals are introduced. The two blue lines at $\lambda = .4565\,\mu$ and $\lambda = .4600\,\mu$, to which the metal cæsium owes its name, are easily discernible in Fig. 9, Plate III., although of feeble intensity. On the other hand, the two red rubidium lines, the deep red color of which has given the name rubidium to this metal, have, as regards energy, a maximum intensity.

Inasmuch as in the cæsium spectrum indications are found of the presence of lines which obviously belong to the metal rubidium, for instance the group at $\lambda = .4200\,\mu$ and $\lambda = .4230\,\mu$, it is probable that in the cæsium chloride used a quantity of the corresponding rubidium salt was present as an impurity. The infra-red cæsium spectrum, as given here, may therefore be considerably affected by lines due to rubidium.

In their investigations,[1] Kayser and Runge have undertaken an approximate estimation of the intensity of many of the lines, and have introduced for this purpose a scale of luminosity consisting of six numbers. Since it may be assumed in the present case that with the possible exception of the violet end of the spectrum, the selective absorption of the apparatus plays no important part, it has seemed allowable in this investigation to regard the intensities of the lines as proportional to the galva-

[1] *Kayser* und *Runge*, Wied. Ann., 41, p. 306, 1890.

nometer deflections. It is to be observed, however, that the more distant infra-red lines appear by this method too weak in intensity, due to the defective achromatism mentioned above.

For all this, the writer does not consider himself justified in introducing a correction in these figures which would thus apply to the intensity of but a few lines at most, and even here the correction is probably not large. In the following tables, in addition to the wave-lengths of the several lines, are added their respective intensities, as read directly from the ordinates of the curves, Figs. 5–9, Plates II. and III. While, indeed, the unit of the energy is an arbitrary one, yet the lines of all the metals examined were measured in terms of the same standard, since the current in the lamp and the sensitiveness of the measuring apparatus were subjected to but feeble fluctuations.

TABLES OF WAVE-LENGTHS AND INTENSITIES.

TABLE X.

LITHIUM.

λ	I	λ	I	λ	I
0.3913 μ	10	0.4615 μ	331	0.670 μ	1191
0.4140 "	58	0.4990 "	34	0.811 "	296
0.4238 "	11	0.6102 "	570	1.800(?)"	(?)
0.4288 "	10				

TABLE XI.

POTASSIUM.

λ	I	λ	I	λ	I
0.4045 μ	30	0.643 μ	11	1.086 μ	108
0.4233 "	16	0.691 "	74	1.155 "	395
0.5113 "	5	0.768 "	1443	1.220 "	205
0.5340 "	9	0.840 "	18	1.470 "	70
0.5362 "	6	0.885 "	13	1.500(?)"	50
0.5800 "	14	0.950 "	23		

TABLE XII.

SODIUM.

λ	I	λ	I	λ	I
0.3932 μ	31	0.5892 μ	877	0.770 μ	22
0.3967 "	31	0.616 "	91	0.818 "	660
0.4236 "	42	0.644 "	22	0.855 "	18
0.4677 "	11	0.671 "	26	0.930 "	8
0.4996 "	62	0.699 "	7	0.995 "	10
0.5164 "	12	0.710 "	14	1.075 "	13
0.5271 "	16	0.714 "	13	1.132 "	419
0.5600 "	19	0.720 "	12	1.245 "	30
0.5685 "	186	0.736 "	10	1.800(?)"	(?)

TABLE XIII.

RUBIDIUM.

λ	I	λ	I	λ	I
0.4200 μ	6	0.627 μ	23	0.945 μ	10
0.4230 "	8	0.669 "	11	0.997 "	151
0.5215 "	5	0.726 "	21	1.063 "	11
0.5270 "	6	0.737 "	19	1.090 "	13
0.5367 "	4	0.775 "	414	1.153 "	26
0.5435 "	4	0.791 "	443	1.224 "	13
0.5592 "	9	0.821 "	42	1.318 "	198
0.5710 "	7	0.845 "	50	1.475 "	102
0.607 "	8	0.878 "	60	1.520 "	71
0.616 "	12	0.913 "	11		

Table XIV.

CÆSIUM.

λ	I	λ	I	λ	I
0.4200(?)μ	4	0.646 μ	10	0.900 μ	155
0.4230(?)"	7	0.674 "	46	0.995 "	182
0.4565 "	15	0.694 "	63	1.150 "	9
0.4600 "	6	0.721 "	47	1.205 "	7
0.5528 "	7	0.775 "	175	1.323 "	81
0.5635 "	7	0.790 "	107	1.420(?)"	38
0.5828 "	28	0.833 "	297	1.450 "	52
0.6010 "	14	0.865 "	151	1.520 "	20
0.619 "	23	0.882 "	345	1.575 "	8
0.629 "	12				

The previously mentioned investigations of Becquerel on the wave-lengths of the infra-red sodium and potassium lines gave values which are compared in the following table with the results of the present observations.

Table XV.

	Becquerel.		Snow.
	With the Prism.	With the Grating.	With the Prism.
Na	λ = 0.819 μ	λ = 0.819 μ	λ = 0.818 μ
	λ = 1.098 "	λ = 1.142 "	λ = 1.132 "
K	λ = 0.770 "	λ = 0.770 "	λ = 0.768 "
	λ = 1.003 "		
	λ = 1.073 "	λ = 1.098 "	λ = 1.086 "
	λ = 1.125 "	λ = 1.162 "	λ = 1.155 "
	λ = 1.182 "	λ = 1.233 "	λ = 1.220 "

Becquerel's measurements with the aid of the prism are observed to be in general smaller than the results here presented, while the latter coincide more nearly with his determinations with the grating. More confidence was placed by Becquerel himself in

the grating measurements, so that the better agreement between his values and those here given may be regarded as a partial confirmation of the determinations as given in this paper.

VIII. CONCLUSION.

This investigation was originally begun to test experimentally in the infra-red the empirical formulæ established by Kayser and Runge for the wave-length of lines in the visible and ultra-violet spectrum.

At the writer's request, Professor Runge very kindly furnished a table of the wave-lengths as calculated by himself and Professor Kayser. These values as given in the following table may be in error by an amount from 10 to 20 $\mu\mu$ on account of the extensive extrapolation demanded.

TABLE XVI.

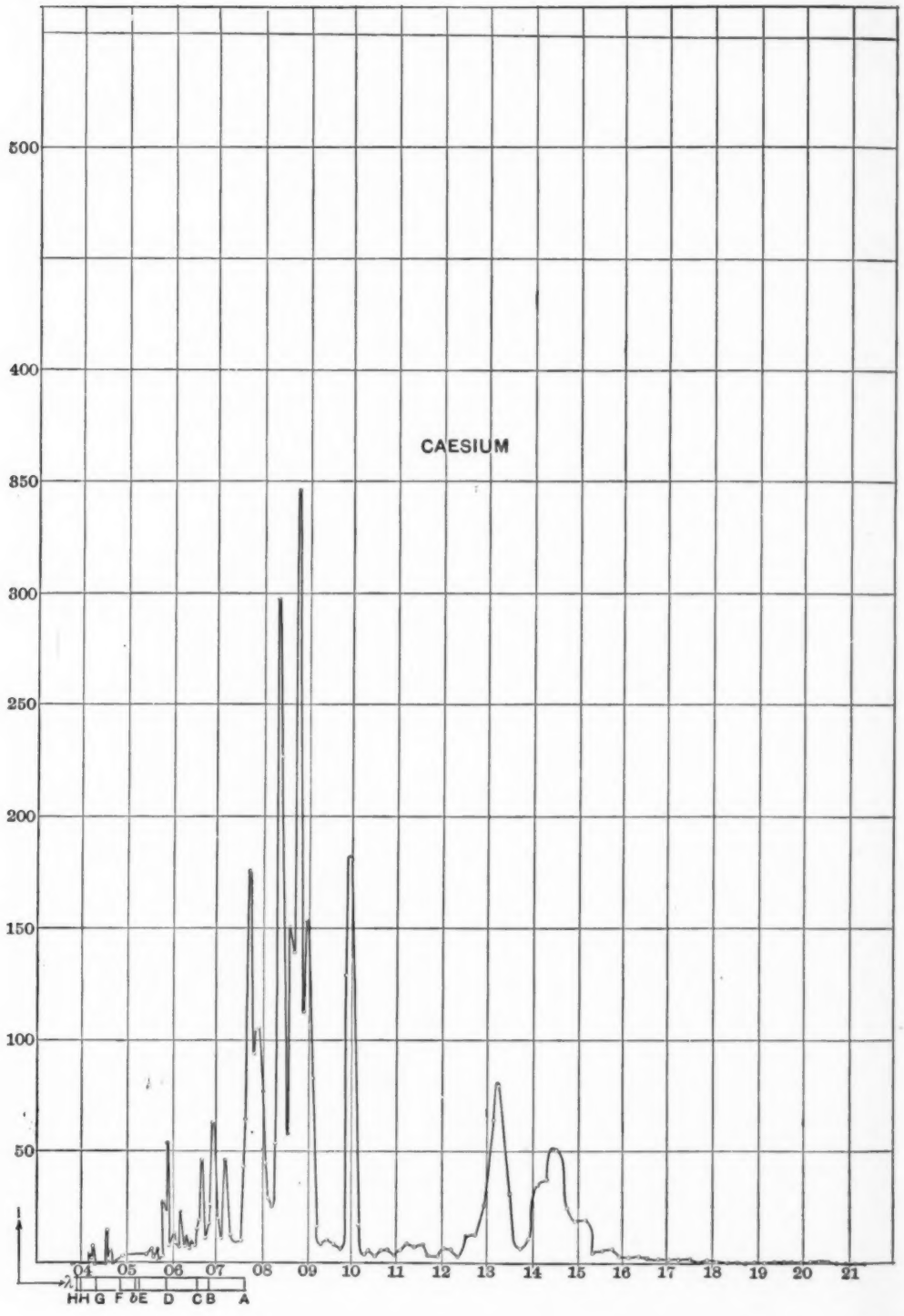
	λ			
Li	0.819 μ			
Na	1.150	1.148 μ		
K	1.266	1.257	1.253	1.244 μ
Rb	1.718	1.653 μ		
Cs	0.922	0.877	0.867	0.828 μ

In a number of cases, for instance with lithium and sodium, the coincidence with the values here obtained is as close as could have been expected. With the other metals, however, noticeably greater discrepancies occur, so that these observations in their present form cannot be regarded as a full confirmation of the formulæ of Kayser and Runge.

I desire in conclusion to express to Professor August Kundt my warmest thanks, not only for the cordial support he so freely gave, but also for his many suggestions and for the kindly interest which he took in the entire investigation.

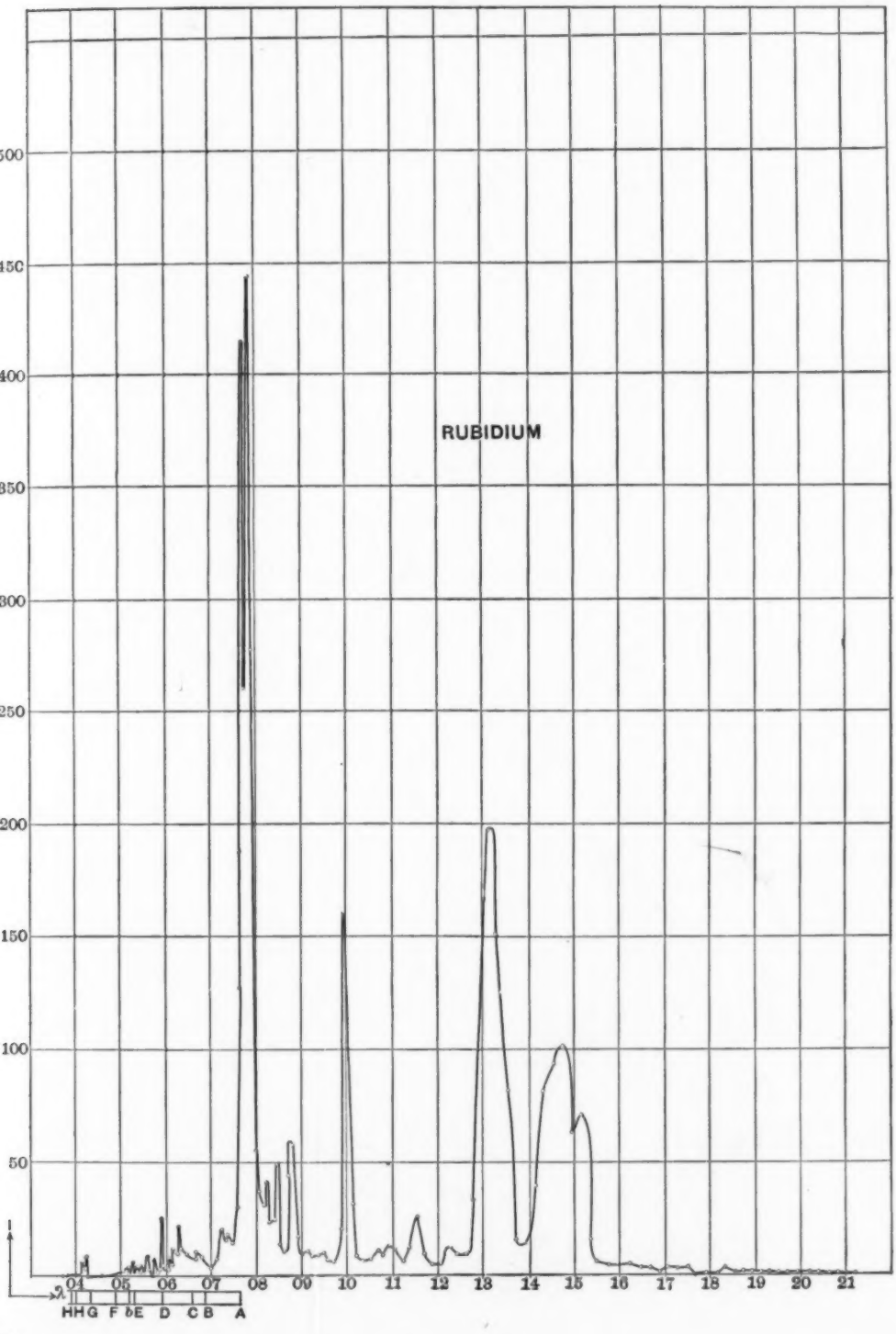
PHYSICAL LABORATORY, UNIVERSITY OF BERLIN,
June, 1892.

Fig. 8.


CAESIUM
500
400
350
300
250
200
150
100
50
I
λ
04 05 06 07 08 09 10 11 12 13 14 15 16 17 18 19 20 21
HH G F δE D C B A

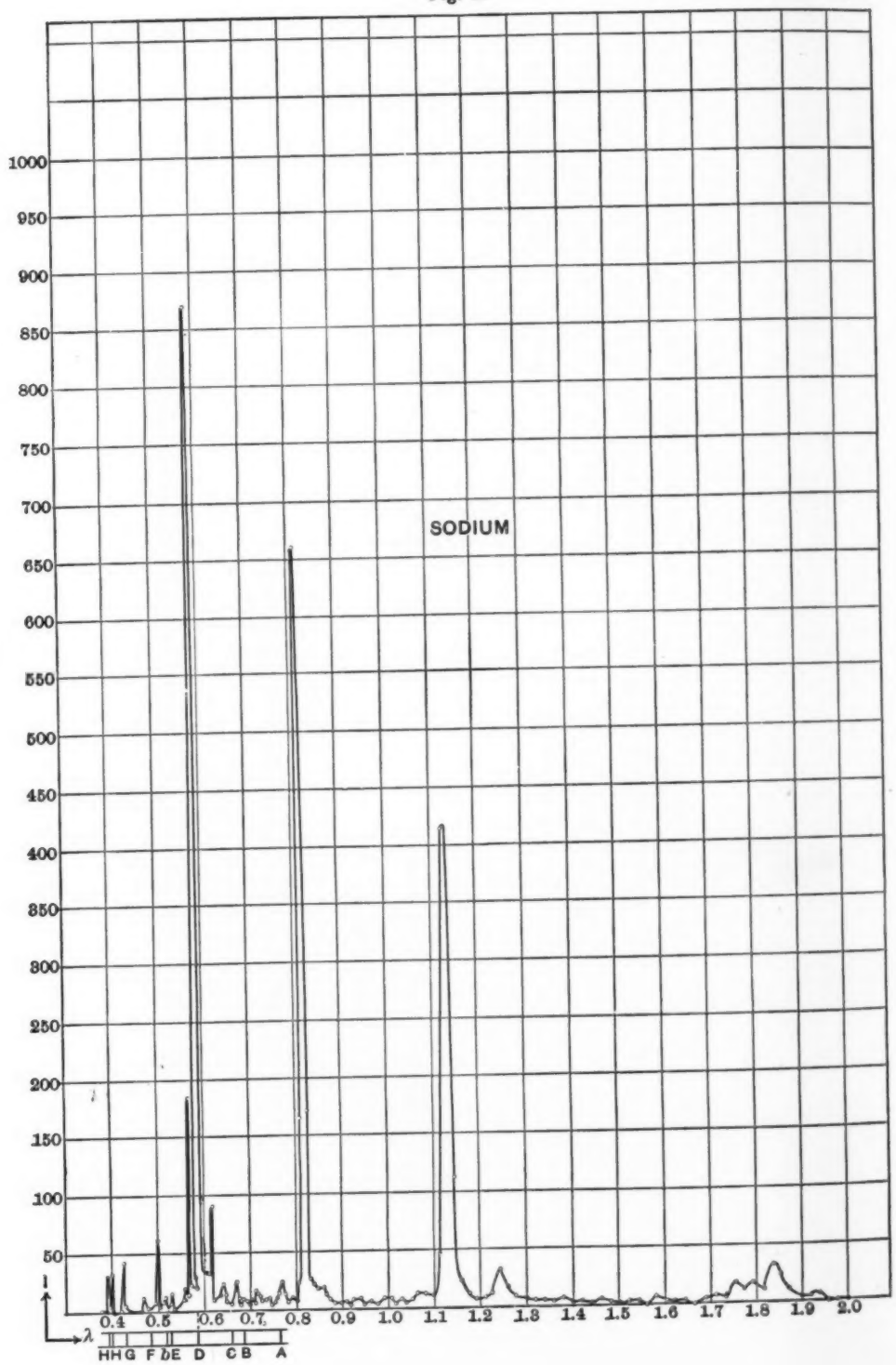
B. W. SNOW.

Fig. 9. PLATE III.



W.

Fig. 5.


SODIUM
1000
950
900
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300
250
200
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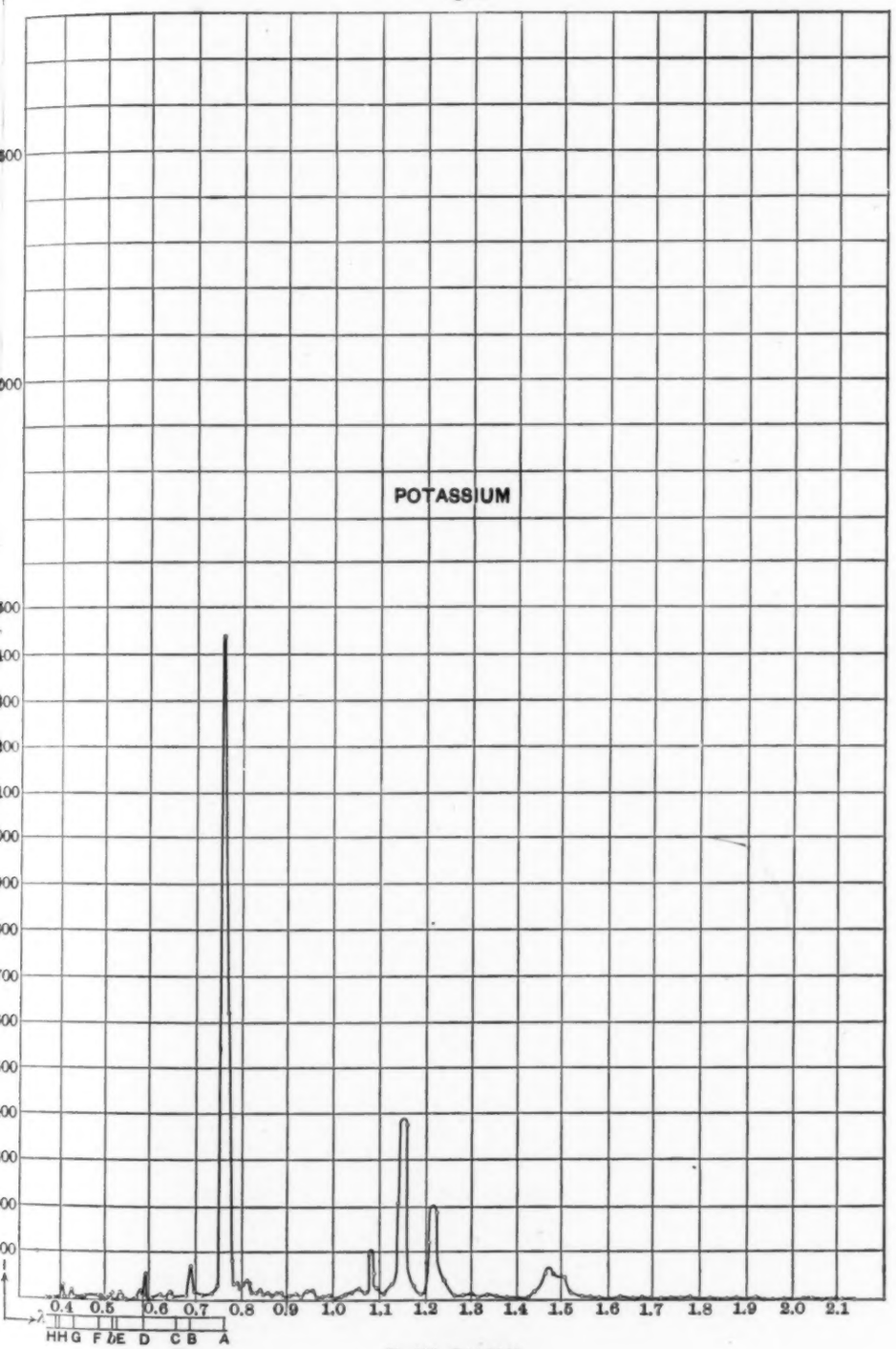
100
50
I
λ
0.4 0.5 0.6 0.7 0.8 0.9 1.0 1.1 1.2 1.3 1.4 1.5 1.6 1.7 1.8 1.9 2.0
H H G F D E D C B A

Fig. 6.



B. W. SNOW.

Fig. 7.

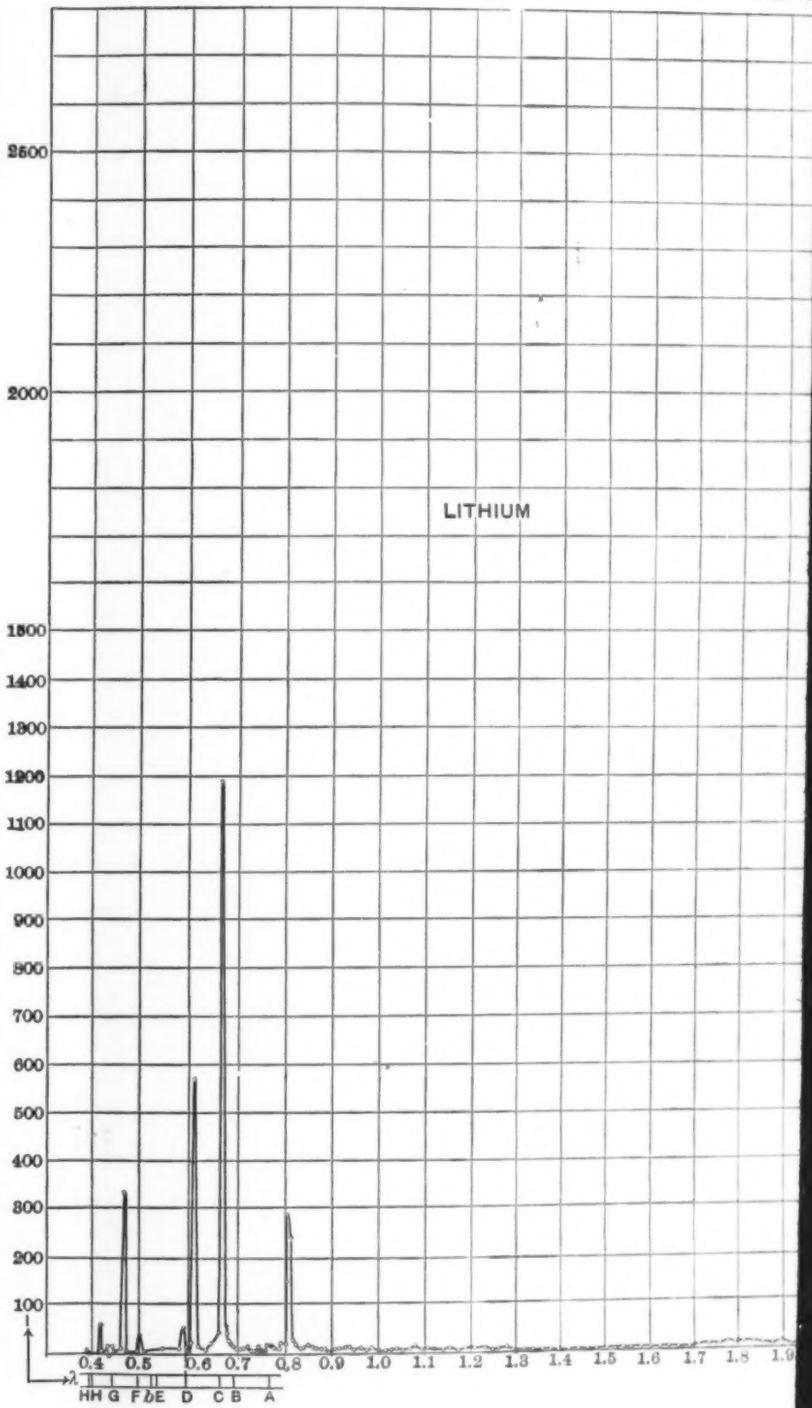
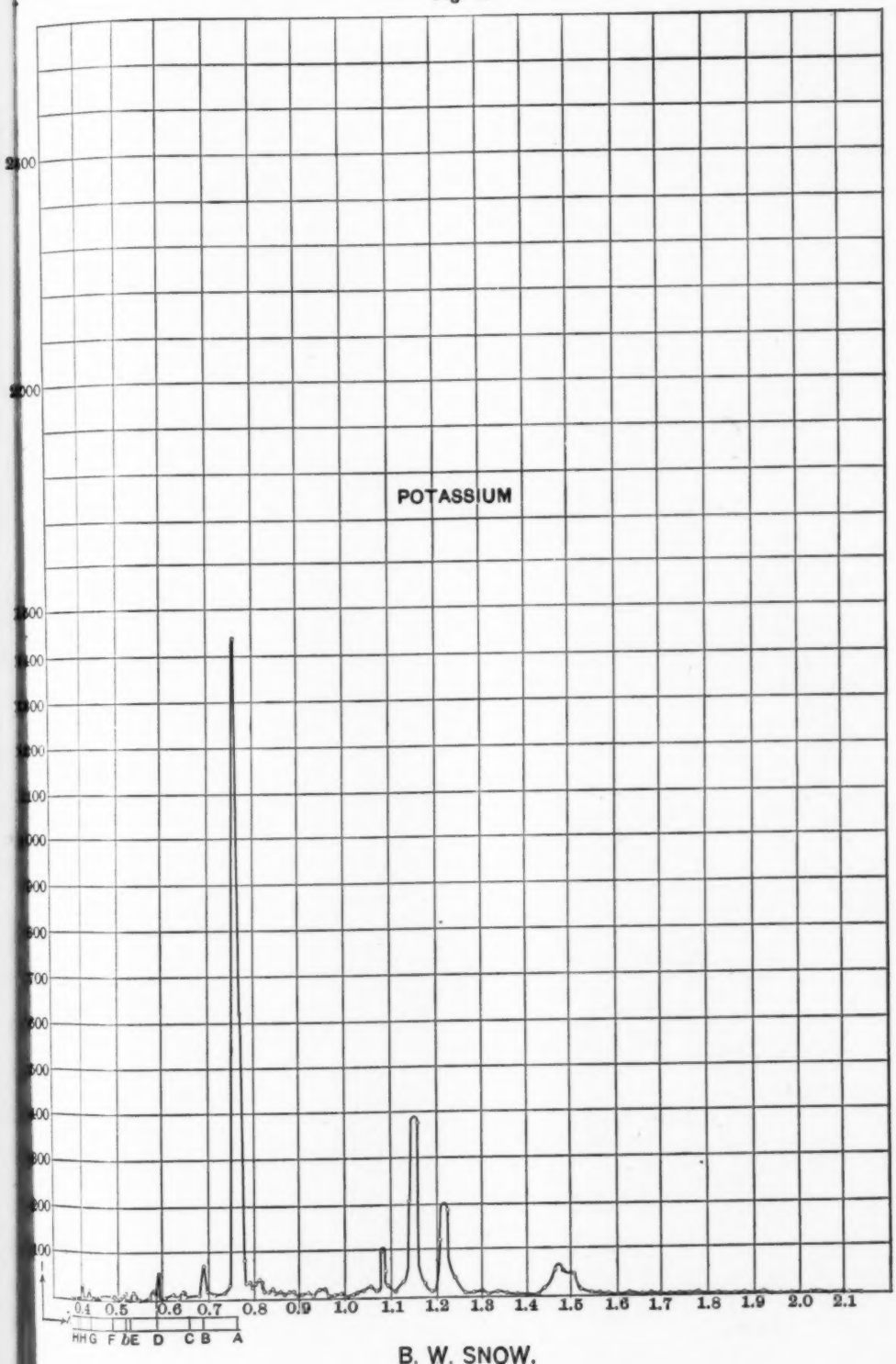
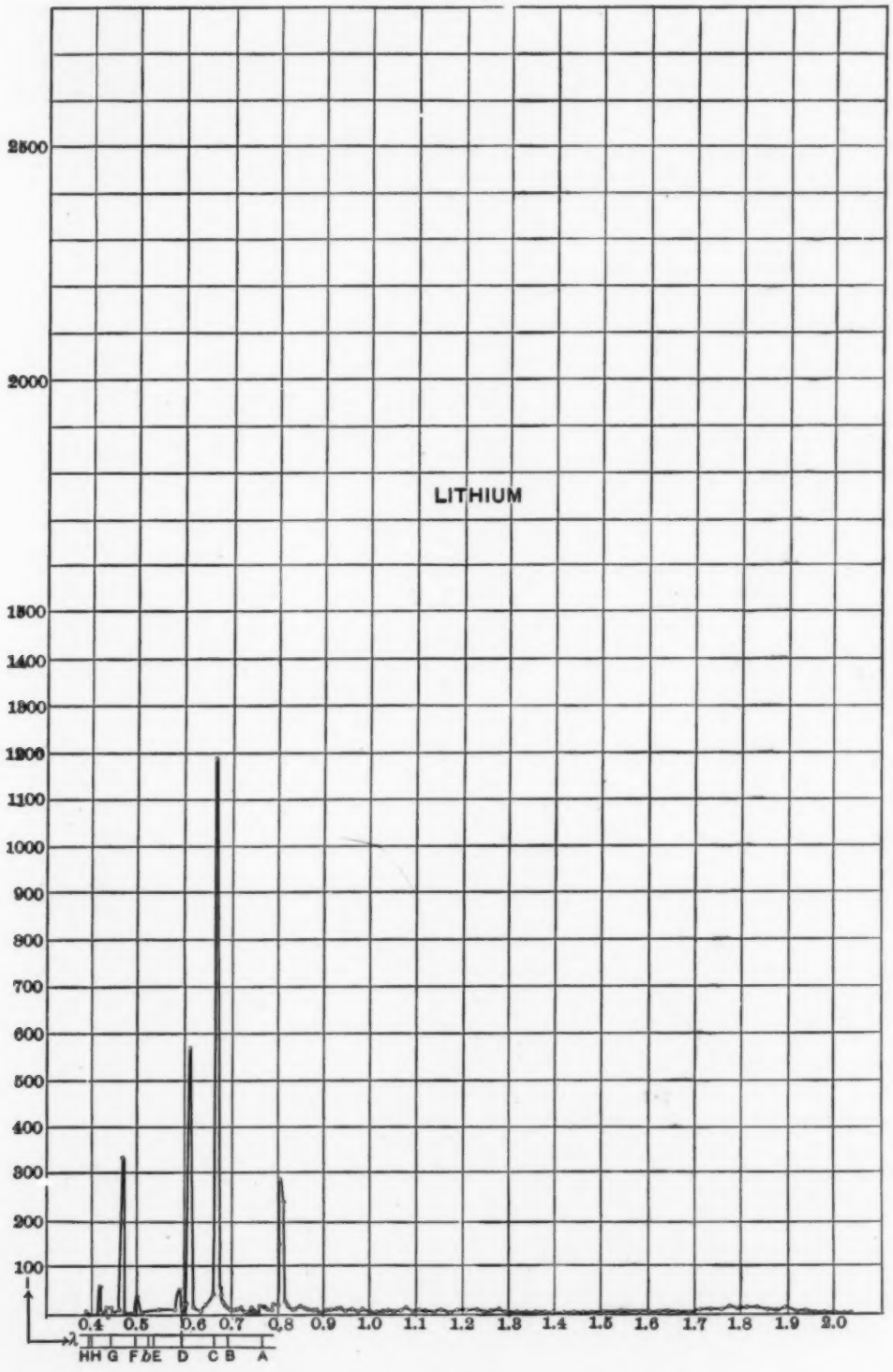


Fig. 6.


POTASSIUM
0.4 0.5 0.6 0.7 0.8 0.9 1.0 1.1 1.2 1.3 1.4 1.5 1.6 1.7 1.8 1.9 2.0 2.1
MMG F E D C B A

B. W. SNOW.

Fig. 7. PLATE II.



GENERAL DISCUSSION OF THE CURRENT FLOW IN TWO MUTUALLY RELATED CIRCUITS CONTAINING CAPACITY.[1]

By Frederick Bedell and Albert C. Crehore.

IN a paper presented at the General Meeting of the American Institute of Electrical Engineers[2] at Chicago, June 7, 1892, the problem of finding the current which flows in a single circuit having resistance, self-induction, and capacity in series was considered. A general solution was obtained for the current which flows when the impressed electromotive force is any function of the time whatever. This solution was applied to four cases, which arise according to the nature of the particular electromotive force considered, and curves were drawn to illustrate the current in each case in the particular yet representative examples assumed.

It is our present object to study the more general case of two independent circuits, each of which contains resistance, self-induction, and capacity, which are connected only by means of their common magnetic field.

Throughout the discussion the following symbols will be used:

R = resistance.

L = coefficient of self-induction (a constant).

M = coefficient of mutual induction (a constant).

C = capacity.

E = (a) constant electromotive force; or

(b) maximum value of harmonic electromotive force.

[1] A paper presented before the World's Congress of Electricians, Chicago, Ill., August, 1893.

[2] Transactions A. I. E. E., Vol. IX., p. 303.

e = instantaneous value of electromotive force.

I = (a) constant current; or

(b) maximum value of harmonic current.

i = instantaneous value of current.

Q = constant charge of condenser.

q = instantaneous value of charge of condenser.

$\omega = 2\pi n = 2\pi \times$ frequency = angular velocity.

$+\theta$ = angle of advance.

$-\theta$ = angle of lag.

T = (a) period; or (b) time-constant.

$j = \sqrt{-1}$.

ϵ = base of Naperian logarithms = 2.71828.

c = arbitrary constant of integration.

The meaning of other letters when used will be evident.

It has been shown that the differential equation of energy for a single circuit is

$$(1) \qquad eidt = \frac{idt\int idt}{C} + Ri^2dt + Li\frac{di}{dt}dt,$$

in which the first member represents the total energy imparted to the circuit by the impressed electromotive force. The second member consists of three terms which represent respectively the three ways in which the energy is used in the circuit. The first term is the energy required to charge the condenser; the second is that expended in heat; the third is that required to produce the magnetic field.

If a second circuit is now placed in mutual relation to the first, so that they possess a common magnetic field, a fourth term must be added to equation (1) depending upon the coefficient of mutual induction of the two circuits. The equation of energy for the primary coil may be written,

$$(2) \qquad ei_1dt = \frac{i_1dt\int i_1dt}{C_1} + R_1i_1^2dt + L_1i_1\frac{di_1}{dt}dt + Mi_1\frac{di_2}{dt}dt,$$

where the subscripts one and two refer to the first and second coil or to the "primary" and "secondary" respectively. The equation of energy for the second coil which has no impressed electromotive force is

$$(3) \qquad 0 = \frac{i_2 dt \int i_2 dt}{C_2} + R_2 i_2^2 dt + L_2 i_2 \frac{di_2}{dt} dt + M i_2 \frac{di_1}{dt} dt.$$

If we divide equation (2) by $i_1 dt$ and equation (3) by $i_2 dt$, we obtain equations of electromotive forces

$$(4) \qquad e = f(t) = \frac{\int i_1 dt}{C_1} + R_1 i_1 + L_1 \frac{di_1}{dt} + M \frac{di_2}{dt};$$

$$(5) \qquad 0 = \frac{\int i_2 dt}{C_2} + R_2 i_2 + L_2 \frac{di_2}{dt} + M \frac{di_1}{dt}.$$

In these equations the electromotive force is expressed as a function of the time. The solution of these differential equations will be the complete solution of the problem of circuits with constant coefficients of mutual and self induction, having resistance and capacity, and it will give us the current that flows in either the primary or secondary at any time due to any impressed electromotive force. In order to solve the equations we may eliminate from the pair either i_1 or i_2 and obtain a single equation containing but one independent variable. To perform this elimination we may differentiate (4) and (5) enough times to obtain one more equation than we have variables to eliminate. We may then form a determinant of the coefficients of the variables to be eliminated and proceed in the usual way in elimination by determinants.

If we use the symbols $D = \frac{d}{dt}$, and $D^2 = \frac{d^2}{dt^2}$, etc., equations (4) and (5) may be written

$$(6) \qquad f(t) = \frac{\int i_1 dt}{C_1} + R_1 i_1 + L_1 D i_1 + M D i_2;$$

$$(7) \qquad 0 = \frac{\int i_2 dt}{C_2} + R_2 i_2 + L_2 D i_2 + M D i_1.$$

By differentiation we obtain the six equations

$$(8) \qquad f'(t) = \frac{1}{C_1} i_1 + R_1 D i_1 + L_1 D^2 i_1 + M D^2 i_2.$$

$$(9) \qquad f''(t) = \frac{1}{C_1} D i_1 + R_1 D^2 i_1 + L_1 D^3 i_1 + M D^3 i_2.$$

$$(10) \qquad f'''(t) = \frac{1}{C_1} D^2 i_1 + R_1 D^3 i_1 + L_1 D^4 i_1 + M D^4 i_2.$$

$$(11) \qquad 0 = \frac{1}{C_2} i_2 + R_2 D i_2 + L_2 D^2 i_2 + M D^2 i_1.$$

$$(12) \qquad 0 = \frac{1}{C_2} D i_2 + R_2 D^2 i_2 + L_2 D^3 i_2 + M D^3 i_1.$$

$$(13) \qquad 0 = \frac{1}{C_2} D^2 i_2 + R_2 D^3 i_2 + L_2 D^4 i_2 + M D^4 i_1.$$

In these six equations there are five quantities which it is required to eliminate, viz., i_1, Di_1, D^2i_1, D^3i_1, and D^4i_1, in order to obtain an equation containing only i_2 and the derivatives of i_2. To obtain a single equation containing i_1 alone, it is necessary to eliminate i_2, Di_2, D^2i_2, D^3i_2, and D^4i_2.

To obtain the equation for primary current, we form a determinant consisting of six columns, five of which contain the coefficients of i_2, Di_2, D^2i_2, etc., in the equations, while the sixth column contains all quantities in the equations independent of i_2, Di_2, or D^2i_2, etc. Thus, to eliminate i_2, we may write the determinant,

(14)

i_2	Di_2	D^2i_2	D^3i_2	D^4i_2	REMAINING TERMS.
0	0	M	0	0	$\frac{1}{C_1} i_1 + R_1 D i_1 + L_1 D^2 i_1 - f'(t)$
0	0	0	M	0	$\frac{1}{C_1} D i_1 + R_1 D^2 i_1 + L_1 D^3 i_1 - f''(t)$
0	0	0	0	M	$\frac{1}{C_1} D^2 i_1 + R_1 D^3 i_1 + L_1 D^4 i_1 - f'''(t)$
$\frac{1}{C_2}$	R_2	L_2	0	0	$M D^2 i_1$
0	$\frac{1}{C_2}$	R_2	L_2	0	$M D^3 i_1$
0	0	$\frac{1}{C_2}$	R_2	L_2	$M D^4 i_1$

$= 0.$

Since the first column is composed entirely of zeros except the fourth row, the determinant reduces to $\frac{1}{C_2}$ times its minor determinant. One column of this minor determinant is all zeros except one row, so that the determinant reduces to one of the fourth order. By erasing the first and second columns and the fourth and fifth rows of the determinant of the sixth order, it becomes

$$(15)\qquad \begin{vmatrix} M & 0 & 0 & \frac{1}{C_1}i_1 & +R_1Di_1+L_1D^2i_1-f'(t) \\ 0 & M & 0 & \frac{1}{C_1}Di_1 & +R_1D^2i_1+L_1D^3i_1-f''(t) \\ 0 & 0 & M & \frac{1}{C_1}D^2i_1 & +R_1D^3i_1+L_1D^4i_1-f'''(t) \\ \frac{1}{C_2} & R_2 & L_2 & & MD^4i_1 \end{vmatrix} = 0.$$

Multiplied out and expanded, this determinant gives

$$(16)\quad \left[(L_1L_2-M^2)D^4+(R_1L_2+R_2L_1)D+\left(\frac{L_1}{C_2}+\frac{L_2}{C_1}+R_1R_2\right)D^2 + \left(\frac{R_1}{C_2}+\frac{R_2}{C_1}\right)D+\frac{1}{C_1C_2}\right]i_1=\frac{1}{C_2}f'(t)+R_2f''(t)+L_2f'''(t).$$

Here we have a differential equation of the fourth order containing but two variables i_1 and t, the solution of which gives the desired primary current in terms of the impressed electromotive force.

The differential equation for the secondary current may be obtained in a similar manner by eliminating i_1 and its derivatives from equations (8) to (13) inclusive. In order to perform the elimination, we may write as before, the determinant

$$(17)\qquad \begin{array}{|ccccc|l|} \hline i_1 & Di_1 & D^2i_1 & D^3i_1 & D^4i_1 & \text{Remaining Terms.} \\ \hline \frac{1}{C_1} & R_1 & L_1 & 0 & 0 & MD^2i_2-f'(t) \\ 0 & \frac{1}{C_1} & R_1 & L_1 & 0 & MD^3i_2-f''(t) \\ 0 & 0 & \frac{1}{C_1} & R_1 & L_1 & MD^4i_2-f'''(t) \\ 0 & 0 & M & 0 & 0 & \frac{1}{C_2}i_2+R_2Di_2+L_2D^2i_2 \\ 0 & 0 & 0 & M & 0 & \frac{1}{C_2}Di_2+R_2D^2i_2+L_2D^3i_2 \\ 0 & 0 & 0 & 0 & M & \frac{1}{C_2}D^2i_2+R_2D^3i_2+L_2D^4i_2 \\ \hline \end{array} = 0.$$

Since the first column of this determinant is composed entirely of zeros except the first row, it reduces to the minor of the fifth order found by erasing the first row and first column. Since the first column of this minor determinant is composed entirely of zeros except its first row, the whole reduces to the determinant of the fourth order found by striking out both the first and second rows and columns. Thus,

$$(18)\qquad \begin{vmatrix} \frac{1}{C_1} & R_1 & L_1 & MD^4i_2 - f'''(t) \\ M & 0 & 0 & \frac{1}{C_2}i_2 + R_2Di_2 + L_2D^2i_2 \\ 0 & M & 0 & \frac{1}{C_2}Di_2 + R_2D^2i_2 + L_2D^3i_2 \\ 0 & 0 & M & \frac{1}{C_2}D^2i_2 + R_2D^3i_2 + L_2D^4i_2 \end{vmatrix} = 0.$$

Multiplying out and expanding, we obtain the desired differential equation for the secondary current.

$$(19)\qquad \left[(L_1L_2 - M^2)D^4 + (R_1L_2 + R_2L_1)D^3 + \left(\frac{L_1}{C_2} + \frac{L_2}{C_1} + R_1R_2\right)D^2 + \left(\frac{R_1}{C_2} + \frac{R_2}{C_1}\right)D + \frac{1}{C_1C_2}\right]i_2 = -Mf'''(t).$$

It is noticeable that these differential equations (16) and (19) for the primary and secondary currents are very similar. The first members are alike if we write i_2 for i_1. The second members show a marked difference. The equation for primary current contains R_2, L_2, and C_2, with the three derivatives of $f(t)$, while the equation for secondary current contains only M, with the third derivative of $f(t)$.

The general solutions of these equations of the fourth order, (16) and (19), would give the value of the primary and secondary currents at any time, where the impressed electromotive force is any function whatsoever of the time, and they would involve the literal solution of the general bi-quadratic equation, that we might find four factors of the bi-quadratic expression in order to resolve the inverse operator into four partial fractions. In their full generality, the equations are too cumbersome for practical purposes, but readily admit of solution, as they now stand, if we

assume the impressed electromotive force to be harmonic, as will be discussed later in this paper. However, the solutions may be obtained for any impressed electromotive force whatsoever, by the introduction of certain modifications which reduce the equations to an order lower than the fourth. If the equation can be reduced to the second order, it is comparatively easy to find its general solution ; but we will not attempt to write the solution of equations of the third and fourth orders, except in the case of an harmonic impressed electromotive force, which is to be discussed later.

If we omit one of the condensers from the circuit, it will reduce the equations (16) and (19) to the third order, and if we omit both condensers, the equations reduce to the second order. The resulting equations in these particular cases may be written down from (16) and (19) by omitting terms and reducing the accents and the order by the required amount. To see how the order is reduced, we will form the equation in the case where both condensers are omitted. To do this let us return to (6) and (7). Omitting the condenser terms, these equations of electromotive forces become

$$f(t) = R_1 i_1 + L_1 D i_1 + M D i_2. \tag{20}$$

$$0 = R_2 i_2 + L_2 D i_2 + M D i_1. \tag{21}$$

We need differentiate each equation only once to obtain enough equations to eliminate either of the variables i_1 or i_2.

By differentiation we have

$$f'(t) = R_1 D i_1 + L_1 D^2 i_1 + M D^2 i_2. \tag{22}$$

$$0 = R_2 D i_2 + L_2 D^2 i_2 + M D^2 i_1. \tag{23}$$

Forming a determinant of these four equations to eliminate i_2, we have

(24)

i_2	Di_2	$D^2 i_2$	Remaining Terms.
0	M	0	$R_1 i_1 + L_1 D i_1 - f(t)$
0	0	M	$R_1 D i_1 + L_1 D^2 i_1 - f'(t)$
R_2	L_2	0	$M D i_1$
0	R_2	L_2	$M D^2 i_1$

$= 0.$

Since the first column is all zeros except the third row, this determinant reduces to

$$(25)\qquad \begin{vmatrix} M & 0 & R_1 i_1 + L_1 Di_1 - f(t) \\ 0 & M & R_1 Di_1 + L_1 D^2 i_1 - f'(t) \\ R_2 & L_2 & MD^2 i_1 \end{vmatrix} = 0;$$

and this expanded gives

$$(26)\quad [(L_1L_2 - M^2)D^2 + (R_1L_2 + R_2L_1)D + R_1R_2]i_1 = R_2 f(t) + L_2 f'(t).$$

Forming the determinant to eliminate i_2, we have

$$(27)\qquad \begin{array}{cccc} i_1 & Di_1 & D^2 i_1 & \text{REMAINING TERMS.} \\ \hline R_1 & L_1 & 0 & MDi_2 - f(t) \\ 0 & R_1 & L_1 & MD^2 i_2 - f'(t) \\ 0 & M & 0 & R_2 i_2 + L_2 Di_2 \\ 0 & 0 & M & R_2 Di_2 + L_2 D^2 i_2 \end{array} = 0.$$

This reduces to

$$(28)\qquad \begin{vmatrix} R_1 & L_1 & MD^2 i_2 - f'(t) \\ M & 0 & R_2 i_2 + L_2 Di_2 \\ 0 & M & R_2 Di_2 + L_2 D^2 i_2 \end{vmatrix} = 0;$$

and, when expanded, gives

$$(29)\quad [(L_1L_2 - M^2)D^2 + (R_1L_2 + R_2L_1)D + R_1R_2]\, i_2 = -Mf'(t).$$

Upon comparing (26) and (29) with (16) and (19) we see that we might have written both (26) and (29) immediately from the more general forms, by reducing each exponent and accent by two and omitting all terms containing C_1 or C_2.

In a similar manner we may find the equations of the third order from the general by omitting one condenser only. If we omit the primary condenser only, we may write from the general the equation in this particular case by reducing all exponents and accents by unity and omitting terms containing C_1. Thus, omitting C_1 from the primary, we have for the primary current

$$(30)\quad \left[(L_1L_2-M^2)D^3+(R_1L_2+R_2L_1)D^2+\left(\frac{L_1}{C_2}+R_1R_2\right)D+\frac{R_1}{C_2}\right]i_1 = \frac{1}{C_2}f(t)+R_2f'(t)+L_2f''(t).$$

The secondary current equation becomes

$$(31)\quad \left[(L_1L_2-M^2)D^3+(R_1L_2+R_2L_1)D^2+\left(\frac{L_1}{C_2}+R_1R_2\right)D+\frac{R_1}{C_2}\right]i_2 = -Mf''(t).$$

If the secondary condenser is omitted, the equations (16) and (19) become

$$(32)\quad \left[(L_1L_2-M^2)D^3+(R_1L_2+R_2L_1)D^2+\left(\frac{L_2}{C_1}+R_1R_2\right)D+\frac{R_2}{C_1}\right]i_1 = R_2f'(t)+L_2f''(t);$$

and

$$(33)\quad \left[(L_1L_2-M^2)D^3+(R_1L_2+R_2L_1)D^2+\left(\frac{L_2}{C_1}+R_1R_2\right)D+\frac{R_2}{C_1}\right]i_2 = -Mf''(t).$$

Another consideration which will enable us to reduce the order of our equations is that of no magnetic leakage. If we consider that all the lines of induction generated by the primary circuit thread the secondary, it is well known that the product of the coefficients of self-induction equals the square of the coefficient of mutual induction of the two circuits; that is, $L_1L_2=M^2$, or $L_1L_2-M^2=0$. This approximation is so nearly realized in most of the closed magnetic circuit transformers that it is worth while to consider how it simplifies the equations. The coefficient of the terms of the highest order in any of the differential equations yet written is this quantity $L_1L_2-M^2$. If we consider therefore that there is no magnetic leakage in the transformer, we may reduce the general equations (16) and (19) to the third order. It will not be attempted here to write down the general solution of the equation of the third order. However, if we wish to consider the case where only one condenser is in circuit and there is no magnetic leakage, equations (30) to (33) inclusive reduce to the second order and can be readily solved. When there is no con-

denser in either circuit, equations (26) and (29) reduce to very simple forms, being of the first order when the consideration of no magnetic leakage is introduced. Upon the solution of these equations of the first order, in the case of an harmonic electromotive force, may be established the well-known transformer diagram usually built up geometrically by a synthetic process.

It is now proposed to obtain the general solutions of equations (26) and (29) which give the complete solution of the problem of two mutually related circuits without considering condensers, but with no limiting assumptions in regard to the magnetic leakage. Then will be given the solution of the same case simplified by the assumption that there is no magnetic leakage. Lastly, the cases will be considered where there is no magnetic leakage, and one of the circuits contains a condenser.

[*To be concluded.*]

THE STUDY OF THE DISTRIBUTION OF STRAINS BY POLARIZED LIGHT.

By A. Marston.

A FEW months since the writer became interested in the determination of the safe loads for a certain class of members of bridges. There was no good theory as to the distribution of the strains in those members, and it was suggested to him to attempt its study by means of polarized light. The method to be described in this paper was worked out in the course of the investigation then undertaken.

The fact that this method, which is mainly of interest to the physicist, was thus the result of the study of an engineering problem is an interesting illustration of the interdependence of the various professions and sciences. It is this interdependence which makes the writer willing to venture to present the results of his experiments in a physical journal, although he is aware that the apparatus at his command was crude, and that his methods of research were approximate. He is the more willing to do this because the interpretation of the results of these experiments depends wholly upon a few elementary principles of optics, well known to all scientists.

When any transparent isotropic substance, such as glass, is subjected to strain, the relative positions of its particles are changed, and, corresponding to the ellipsoid of elasticity of a bi-refractive crystal, we have the ellipsoid of strain. Hence, we should expect that when light falls upon it, only those components of the vibrations would be transmitted which are parallel to the axes of the diametral section of the ellipsoid of strain made by a plane perpendicular to the ray of light, just as in the case of the bi-refractive crystal.

As a matter of fact this property of glass has long been known. When a plate of it is subjected to strain, and interposed between the polarizer and analyzer brilliant color effects are produced, just as with thin crystalline plates.

This property of glass can be taken advantage of to determine exactly the intensity and direction of the strain at any given point in it. Since glass is an isotropic body, whose elastic constants are known, we are able to test any theoretical solution of the general problem of determining the distribution of the strain in an isotropic body by comparing the results of theory with the strain in a similar body of glass, subjected to the action of similar external forces. In this line something has already been done. Wertheim,[1] whose work is referred to in Tyndall's "Lectures on Light," used the colors of polarization in the study of the elasticity of glass. He established the fact that the retardation of the ray is proportional to the load. He described a device, which he designated the "chromatic dynamometer," for the study of stress uniformly applied. In the Transactions of the American Society of Civil Engineers, Vol. III., Nickerson has published an account of some experiments to determine the position of the neutral axis in beams by this method. The theory on which he based the interpretation of his results was, however, incorrect, and it seems strange that his work was not criticized at once.

There are two things to determine in order to specify the strain at any point in a solid; one, the *directions* of its axes, and the other, the *intensities* of the strains along those axes. Corresponding to these two things we have two sets of effects when a glass body, strained so that one of the axes of strain has a known constant direction for all points of the body, is placed between two Nicol's prisms and examined by polarized light passed through it parallel to the known axis. First, if the body be strained in such a way that the lines of strain vary in direction from point to point, we shall find dark bands traversing the glass body, which will vary in position as the prisms rotate relative to the glass body or to each other. In the second place, we shall observe color bands

[1] Wertheim: Comptes Reudus 32, p. 289 (1851). Also, Phil. Mag. (4) 8, p. 241 (1854).

varying in quality and position, with similar rotation of the prisms. The dark bands depend upon the directions of the lines of strain, while the color effects are due to the relative intensities of those two principal strains whose axes are perpendicular to the direction of light. For the sake of clearness we will consider these effects separately in the discussion which follows, and separate them in the figures illustrating the appearances observed in actual cases.

The Directions of the Lines of Strain.

We will take up first, then, the study of the directions of the lines of strain throughout the body. In this case, the interpretation of the meaning of the dark bands observed in the field of view of the analyzer will be much simplified if we keep the prisms crossed. Let the polarizing plane of the polarizer be set at any given angle with the vertical. Then at all points where the lines of strain are parallel or perpendicular to the plane of polarization, the light will not have the direction of its vibrations changed in passing through the glass, and consequently cannot pass through the analyzer. Hence at those points the surface of the glass will remain dark. At any other point, however, the plane of polarization will be rotated so as to contain a tangent to the line of strain. Light from that point will, therefore, be able to pass more or less completely through the analyzer.

The result will be that the surface of the glass, as seen through the analyzer, will appear to be crossed by one or more dark bands, which will give the loci of the points where the lines of strain are parallel or perpendicular to the plane of polarization of the polarizing prism.

By setting the plane of polarization at any desired angle with the vertical, we may thus determine the direction of the lines of strain at a number of points in the glass body, and may draw elementary arcs through those points. By varying the angle of inclination of the polarizing plane and taking a series of observations, we may obtain the elementary arcs at points as close together as desired. By connecting these elementary arcs, we shall then have an approximate sketch of the lines of strain throughout

the body, and sometimes may be able to deduce at once the true theory of their location.

To illustrate the application of this method let us take the case shown in Plate I. Here a one-inch cube of optical glass was pressed by a one-inch roller of optical glass loaded with 400 lbs. The cube had two opposite faces polished, and the others carefully ground. The polished faces were the ones observed, and were set at right angles to the rays of polarized light. The roller was one inch in diameter and one inch long, and was carefully ground cylindrical. Its ends were polished. It was placed with its axis parallel to the rays of light, and the line of contact was a medial line of the upper surface of the cube. The cube rested on the upper face of a piece of plate glass $\frac{1}{8}''$ thick, $1''$ wide in the direction of the rays of light, and $1\frac{1}{2}''$ long. The plate glass rested on a block of hard wood. The applied force was measured by spring balances.

The dark bands shown in Figs. 1 to 8 were sketched by dividing the observed face of the cube into $\frac{1}{10}''$ squares by placing against it a reticule of fine threads stretched in a brass frame. The sketches were made on one-inch squares similarly divided, as shown in the figures.

The polarizer was a Nicol's prism having the distance between parallel sides about $1\frac{1}{4}''$. The analyzer was a similar prism having about $1''$ between parallel sides. These prisms were set in brass tubes in the usual way, and were mounted on wooden stands in such a way that the plane of polarization of each could be set at any desired angle with the vertical, and the angle measured within a degree or two.

When the strained cube was placed between the polarizer and analyzer, with the plane of polarization inclined to the right of the vertical, the amount indicated on each figure, the dark bands shown on the figures were observed. It is to be noted that in addition to these dark bands, color effects were also observed similar to those shown on Plate II. These are omitted from the figures for reasons already given.

In interpreting the results of this experiment, we notice first that there are two regions, in the right and left upper corners

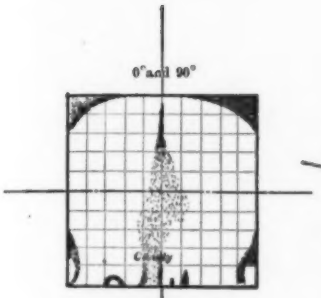


Fig. 1.

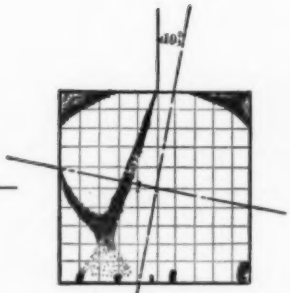


Fig. 2.

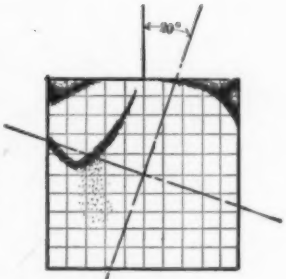


Fig. 3.

PLATE I.

DETERMINATION OF LINES OF STRAIN BY POLARIZED LIGHT.

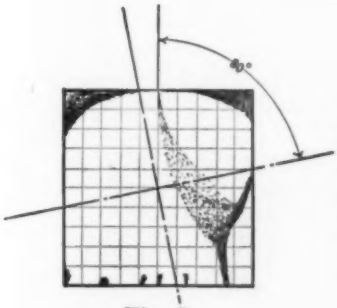


Fig. 8.

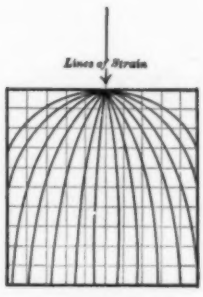


Fig. 9.

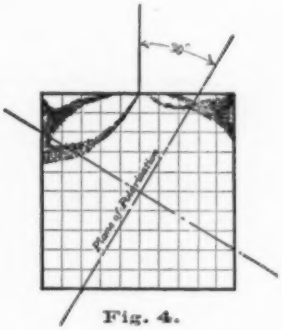


Fig. 4.

1 inch Cube Pressed by 1 inch Roller Loaded with 400 Lbs. The Color Bands are Omitted from the Figures. Prisms Crossed.

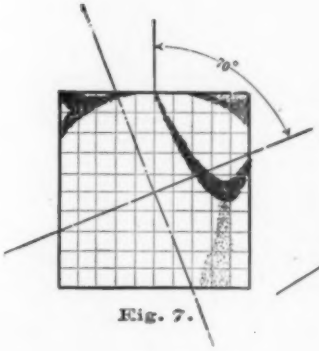


Fig. 7.

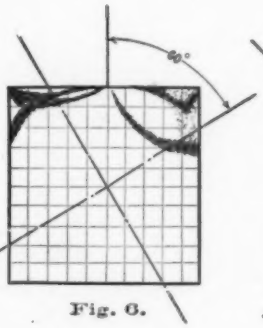


Fig. 6.

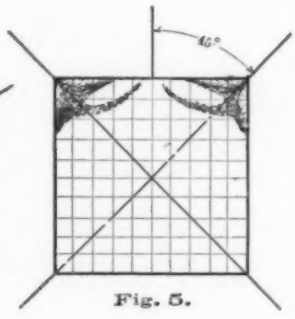


Fig. 5.

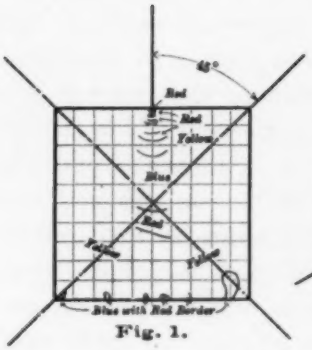


Fig. 1.

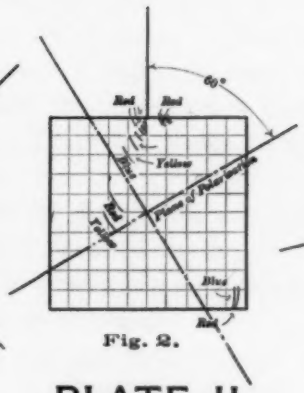


Fig. 2.

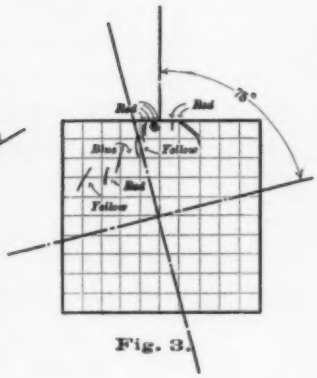
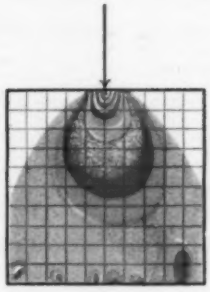


Fig. 3.

PLATE II.

DIFFERENCE OF PRINCIPAL STRAINS BY POLARIZED LIGHT.



NOTE. — There were six bands of red plainly visible, with a seventh probably just on.

Fig. 7.

1 inch Cube Pressed by 1 inch Roller Loaded with 400 Lbs. Prisms Parallel.

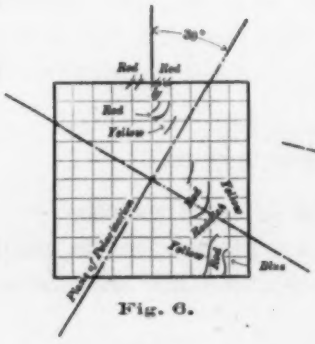


Fig. 6.

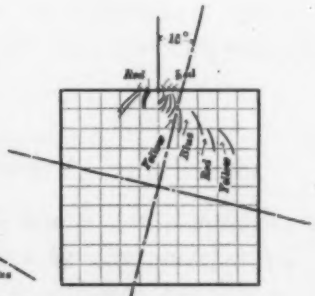


Fig. 5.

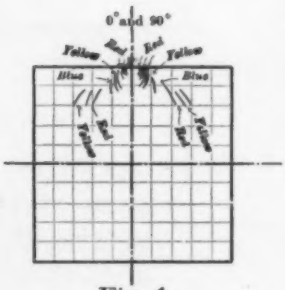


Fig. 4.

respectively, which were dark for nearly all positions of the prisms. In those figures in which these regions are not dark, the light may have been reflected from the sides of the prism in such a way as to produce the appearances seen. It seems probable that the stress was there too feeble to give the glass any bi-refractive qualities.

Aside from this, certain of the dark bands indicate the regions where the lines of strain are inclined in the direction of the planes of polarization, and the others the regions where the lines of strain are perpendicular to those planes.

Where these dark bands, and the dark regions already mentioned in the upper right and left corners overlap, we have the curious appearances seen in Figs. 4, 5, and 6.

By drawing elementary arcs, inclined at the proper angles to the vertical, through the points indicated by the dark bands, and connecting them, we obtain the sketch of the lines of strain shown in Fig. 9.

The dark places along the bottom of the cube, seen in Figs. 1, 2, and 8, indicate points of local intensity of pressure and distortion of the lines of strain, due to the unevenness of the bearing surface. By comparing closely with Fig. 7, Plate II., we can see how the lines of strain converge from each side to these points of local intensity of pressure.

In this case the equations of the lines of strain have not been worked out, but in a series of experiments, which the writer has in progress, he was able to decide at once upon the true theory of their directions. In making a complete study of the theory of the strain in the cube under consideration, it would be necessary to make a set of experiments similar to the one illustrated, for different loads and for different dimensions of rollers and cubes.

The Intensity of the Strain.

Let us consider next the study of the intensity of the strain by means of the color-effects already referred to. It may be said, in advance, that no such complete solution of this problem

can be obtained as of that of the directions of the lines of strain. Nevertheless, we get very interesting and instructive results.

In what follows, it must be understood that the discussion applies only to cases where one of the axes of strain has a known direction, constant for all points of the glass, and that the polarized light is passed through the glass parallel to this known axis.

Reasoning by analogy from bi-refractive crystals, we should expect that the velocities of the two rays of light in the case under consideration would be inversely proportional to those two axes of the strain ellipsoid which are at right angles to the direction of the polarized light, *i.e.* to the intensities of the principal strains in those directions. Actual experiments, whose results will be given hereafter, approximately verify this assumption, at least for small strains. Hence the relative retardation will depend upon the relative intensities of the principal strains named, and in the colors observed in the strained glass we have a measure of the difference between those principal strains at each point of the body.

Hereafter, unless otherwise specified, by principal strains will be meant those two principal strains at right angles to the ray of light.

When the difference between the principal strains at any point in the glass gradually increases from 0, the first color waves in the polarized light to be suppressed will be the shortest ones, *i.e.* those in the vicinity of violet. We should therefore expect that the first color to make its appearance would be the complementary color, yellow, and that following this we would have red, and then blue. When the difference becomes equal to one and one-half times that causing yellow we will again have yellow, the relative retardation in this case being one and one-half times the wave-length of violet. Following this second yellow will come second bands of red and blue, and so on.

To show how completely this theory is borne out by experiment, we give the results obtained by subjecting the glass cube, already experimented with, to strains of known character. In an ordinary Olsen testing machine of 50,000 lbs. capacity, the cube was sub-

jected to a series of uniformly distributed vertical compressive stresses, gradually increasing from 0 to 7050 lbs. The appearance when polarized light was passed through it was observed by means of the Nicol's prisms, already described. The prisms were set with their polarizing planes parallel, and inclined at an angle of 45° to the vertical. The resulting colors are shown in the following table: —

TABLE NO. 1.

SHOWING COLORS OBSERVED IN 1″ CUBE OF OPTICAL GLASS WHEN SUBJECTED TO SIMPLE VERTICAL COMPRESSIVE STRESSES.

Load.	Color.	Load.	Color.
100 lbs.	Clear	2100 lbs.	Greenish-yellow
200 "	Cloudy	2550 "	3d Red
300 "	Faint yellow	3000 "	Green
400 "	Bright yellow	3650 "	4th Red
525 "	1st Red	4050 "	Green
600 to 900 "	Blue	4750 "	5th Red
		5250 "	Green
1200 "	Yellow	5900 "	6th Red
1550 "	2d Red	6450 "	Green
1800 "	Greenish-blue	7050 "	7th Red

In making this experiment it was found that the delicacy of the test was such that it was very difficult to get rid of the effects of local pressures, due to slight inequalities in the bearing surfaces. However, by using steel bearing blocks one inch square, having the bearing surfaces carefully surfaced, and by interposing a few thicknesses of paper between them and the cube, the resulting colors were so evenly distributed over the face of the cube that no doubt was left in the mind of the observer that such slight unevenness as still remained was due entirely to inequalities in the distribution of the applied load.

In this experiment, since both of the principal stresses other than the vertical were 0, the difference between the principal

strains is proportional to the load. The results may be regarded as a calibration of the observed colors for the glass cube when it is subjected to strain.

It was observed that the range of the loads corresponding to any particular band of red was less than for either of the other colors. As near as could be observed the loads corresponding to the middle lines of the different bands of red were as follows:— 1st, 525 lbs.; 2d, 1550 lbs.; 3d, 2550 lbs.; 4th, 3650 lbs.; 5th, 4750 lbs.; 6th, 5900 lbs.; 7th, 7050 lbs.

In interpreting these results it is to be noticed that the loads given are approximate, because each is for a ***band*** of color, which corresponds to a certain *range* of loads. Moreover, the testing machine used was not very sensitive to increments of load less than 100 lbs., and no means were available for testing its accuracy. To remove as far as possible inaccuracies due to the testing machine, a similar series of observations was made on a piece of optical glass one inch thick by two inches square, finished similarly to the glass cube before tested. The loads for the same strains and colors as in the cube should have been twice those already given. The results were as follows:— 1st, 1100 lbs.; 2d, 3175 lbs.; 3d, 5300 lbs.; 4th, 7500 lbs.; 5th, 9600 lbs.; 6th, 12,000 lbs.; 7th, 14,100 lbs.

These results establish the law that, as far as the experiments extended, the retardation varies approximately directly with the strain. They indicate that the stress necessary to produce a retardation of one wave-length of greenish blue, the complementary color of red, is about 1065 lbs.

The experiments could not be carried further because, as the loads increased, the colors became pale, and after the seventh series were hardly distinguishable from each other or from white light. After the second series we have green in place of the yellow and blue. This increasing paleness of the colors is due to the well-known fact that, for retardations of several half wave-lengths, the interference effects in the case of white light become indistinct. This difficulty could be overcome by using monochromatic light.

To determine the effect of compound stress upon the colors, a

small hydraulic press was constructed, by which horizontal pressures as high as 300 lbs. could be applied to the inch cube at the same time that it was subjected to vertical pressure in the Olsen machine. The following will serve as an illustration of the results:

TABLE NO. 2.

SHOWING COLORS OBSERVED WHEN POLARIZED LIGHT IS PASSED THROUGH A 1″ CUBE OF OPTICAL GLASS SUBJECTED TO COMPOUND STRESSES.

Pressures.		Color.	Pressures.		Color.
Vertical.	Horizontal.		Vertical.	Horizontal.	
0	0	Clear	400	300	Nearly clear
100	0	Cloudy	500	0	Blue and red
200	0	Cloudy	500	100	Bright yellow (and a little red at center)
300	0	Faint yellow	500	200	Faint yellow
300	100	Cloudy	500	300	Cloudy
300	200	Cloudy	600	0	Blue
300	300	Clear	600	100	Blue and red
400	0	Bright yellow	600	200	Yellow (red at center)
400	100	Faint yellow	600	300	Yellow
400	200	Cloudy			

Similar observations were made for a series of vertical loads varying up to 4400 lbs. In the above experiments the lack of uniformity in the coloring in different parts of the field indicates some irregularity in the distribution of the loads. Allowing for this, however, the results show just what our theory would lead us to expect, viz. that the colors observed correspond to those caused by simple stresses equal to the *difference* between the components of the compound stress.

To further determine the effect of compound stress, a few experiments were made with a slip of plate glass $1\frac{1}{8}'' \times \frac{5}{16}'' \times 3\frac{1}{2}''$,

which was subjected to vertical tensions in the direction of its length of from 0 to 250 lbs. in the Olsen machine, and at the same time to horizontal pressures by the hydraulic press. The horizontal pressures were applied to the middle of the edges of the slip, over a space about one inch long, and varied from 0 to 300 lbs. The results showed in general that the resulting colors corresponded to simple stresses, equal to the numerical *sum* of the components of the compound stresses.

The preceding experiments show that, for small strains at least, *the colors seen in a strained glass body, when polarized light is passed through it in a direction parallel to one of the axes of strain, are measures of the algebraic differences of the intensities of those two principal strains whose directions are perpendicular to the direction of the polarized light.*

Since the axes of the principal stresses coincide with those of the principal strains, and since the difference of any two principal strains is directly proportional to the difference of the corresponding principal stresses, it follows that the word *stress* might have been substituted for *strain* throughout the preceding statement.

Let us now apply this method of studying the intensity of the stresses to the case of the one-inch cube of optical glass, for which the lines of stress have already been determined. [See Plate I. and the accompanying discussion.]

The manner of applying the vertical load of 400 lbs. to this cube has already been described. The polarizer and analyzer for the experiment were the Nicol's prisms already described. In this case, however, their principal planes were set parallel, so that the cube in its unstrained condition was clear. By this means we get rid of all dark bands such as those shown on Plate I., and the colors make their appearance for smaller loads than do the complementary colors seen when the prisms are crossed.

When we look at the glass cube through the analyzer we see a series of bands of yellow, red, and blue, instead of the uniform tint seen in the preceding experiments. Each of these bands indicates a region where the difference between the intensities of the principal stresses falls within the range of stress shown in Table I. for the corresponding color. We observe, however, that these color

bands are not well defined in all parts of the cube for any one position of the prisms, and that when the prisms are rotated, the region in which the color bands are well defined changes.

To understand the reason for this, we must look at what takes place when the ray of polarized light passes through the glass at any point. As has already been said, it is divided into two rays, polarized in the directions of the axes of the stress. Now if one of the axes of the stress be parallel to the plane of polarization, the ray polarized in the direction of the other axis cannot pass through the analyzer at all, and consequently no interference can take place. On the other hand, when the axes of the stress make angles of 45° with the plane of polarization, the rays pass through the analyzer in equal amounts, and interference is complete. For all other angles than 45°, 0°, or 90° between the axes of stress and the plane of polarization, partial interference can take place. For this reason, in the experiments with uniform simple and compound stresses on the cube, the prisms were set at 45° to the vertical. Since the axes of strain were vertical and horizontal, the angle of 45° gave the position for complete interference. In those experiments the glass became colorless when the prisms were set with their planes of polarization vertical or horizontal, and nearly so for any angle with the vertical differing much from 45°.

In the case at present under consideration, since the lines of stress vary in direction at different points in the cube, in order to map out the color bands throughout their whole extent, it will be necessary to take a series of observations with the prisms set at different angles with the vertical, and to combine the results. This is what was done in the experiments, the results of which are given in Plate II. Fig. 1 shows where the bands were sharply defined when the prisms were set at 45° with the vertical. Evidently this will be where the lines of stress are nearly vertical or horizontal. Similarly, Fig. 2 marks the bands plainly where the lines of stress are inclined 15° or 75° to the vertical, Fig. 3 where they are inclined 30° or 60°, and so on. By comparing with Fig. 9, Plate I., it will be seen how completely the theory is borne out.

By combining the results shown in Figs. 1 to 6, we obtain

the complete outlines of the several color bands as shown in Fig. 7. We thus obtain a map, as it were, of the difference of the intensities of the principal stresses. By the aid of Table I., we can assign an approximate numerical value to this difference at any point of the cube. Thus, at the line of contact between the roller and the cube, the difference between the horizontal and vertical components of the stress is about 7000 lbs. per square inch, and since both the stresses are compressive at this point, the maximum pressure between the roller and the cube must be greater than 7000 lbs. per square inch, though the vertical load was only 400 lbs.

The stresses themselves are in this case probably greater at points where their difference is greater, so that Fig. 7 gives in a general way an idea of the distribution of the stress throughout the cube. Thus, a quarter of an inch from the bottom we see that the stress is quite uniformly distributed throughout the whole cube. The fact that the color here is bright yellow, like that caused by a uniformly distributed load of 400 lbs. [see Table No. I.], shows that the horizontal stresses there are nearly zero.

The small areas of red and blue along the bottom of the cube show points of local intensity of pressure, due to inequalities in the bearing surfaces. It is evident that, for some reason, the pressure on the right side of the cube was greater than elsewhere.

If we were attempting to express the stress at any given point in the cube by a formula, we could make a series of experiments like that whose results are given in Plate II. A careful study of all the results would give checks for any proposed formula, or might suggest an approximate solution of the problem, which would answer for practical purposes.

In making any such series of experiments, the writer believes that it would be practicable, with suitable apparatus, to throw an image of the cube, or other glass body experimented with, upon a screen, on which the dark bands and color bands could then be easily and accurately traced. To obtain the best results, the experiments should be conducted with monochromatic light.

AMES, Iowa, June 12, 1893.

MINOR CONTRIBUTIONS.

Editorial Note.

THE physicist has occasion to give thought to many matters which are of considerable scientific interest, but which, for one reason or another, he does not embody in a memoir. To this class of subjects belong details of laboratory manipulation or demonstration, observations of isolated facts, etc., etc. Phenomena of great interest and beauty, indeed, such as one meets with in the every-day routine of the laboratory, often go undescribed. The contemplation of such phenomena is one of the chief charms in the pursuit of physics, and yet we are apt to forego mention of them, either because they seem to be matters too obvious to deserve publication, or because their relations are obscure.

It is for topics of the classes just indicated that the editors of the *Physical Review* have planned the department of *Minor Contributions*. It is not intended to make it a department of minor interest, nor for the discussion of matters of merely minor importance. It is believed, indeed, that the communications to be grouped under this head will be full of valuable suggestion, that they will be read with much interest, and that writers whose productions appear as "minor contributions" will find themselves in as good company as elsewhere in the *Review*.

It is the fashion in these crowded times, especially among the men of science who write in English, to cultivate a condensed style. All details not essential are omitted, and many of the very features which in the course of the research had most delighted the investigator, and which, if allowed expression, would have been a source of pleasure to his readers, are refused a place.

It is the presence of these minor elements which gives to the memoirs of some of the masters in the early part of the century their charm, and which renders especially attractive the reading of the works of certain of the French physicists even of a later

day. In such side-dissertations, embodied in the classics of physics, we find many an intimation of the discoveries of a later day, many an interesting glimpse into the future of the science. If through this department of the *Review* an opportunity is made and taken advantage of, whereby expression may be had of these minutiæ of physics, which, suggestive and ofttimes important though they be, seem to be in danger of being dropped from our literature, the purpose of the editors will have been fully accomplished.

AN ACOUSTIC PYROMETER.

FERNANDO SANFORD.

IN devising a piece of apparatus for showing the increase of velocity of a sound wave in air for an increase of temperature, the idea occurred to me that the same apparatus could be used for measuring high temperatures in the interior of furnaces or factory chimneys. I remembered also the suggestion of Mr. Preston in the *Philosophical Magazine* of July, 1891, that a resonant tube and tuning fork be used for this purpose, but believing that the apparatus which I had prepared had some advantages over the one proposed by Mr. Preston, I have thought that it might be of sufficient interest to warrant its publication.

The apparatus which I had prepared consists of a brass tube $5\frac{1}{2}$ inches long, of $\frac{5}{8}$-inch bore, closed at one end by a brass plug. To this tube is fastened by a movable clamp another brass tube about a foot long of $\frac{1}{4}$-inch bore, so bent and flattened at the end next to the larger tube that in blowing through it a current of air is driven across the open end of the larger tube, and causes it to give forth a musical note.

Upon heating the sounding tube the pitch of the note rises, increasing with the square root of the absolute temperature. The tube can be immersed almost to its mouth in a liquid, or can be placed entirely within a furnace or chimney with only the mouthpiece projecting, and can be blown by a bellows, or by air from the lungs.

I made a series of measurements with the tube in water at

different temperatures from zero to eighty degrees, determining the pitch of the note by Appun's "*Ton Messer*," and found that my determinations varied by three or four degrees from the temperature of the water as determined by a thermometer. This variation seemed to me to be mostly due to a difference in temperature between the air inside the tube and the water outside. A part of it was due, however, to the coercive power of the reeds in the "*Ton Messer*" upon the column of air in the tube. At the temperature of the experiment a difference of one degree in temperature would make something less than one vibration a second difference in the pitch of the note given by a column of air of this length. With the series of reeds differing by four vibrations a second from each other, a difference of one vibration a second in the pitch of the note could be determined; but when the tube was nearly in unison with the reed it would make only a few beats at starting, and would then vibrate in exact unison with it. This difficulty would be still greater in attempting to use a resonance tube with a tuning fork, as in that case the sound of the fork is reinforced by a tube varying considerably in length from the one giving the greatest reinforcement.

I was unable to compare my measurements made at higher temperatures with any other standard; but I was able to determine the pitch with the same degree of accuracy up to the highest temperature to which I could heat the tube in a gas flame.

It goes without saying that the tube could be as well made of platinum or porcelain for measuring high temperatures, and that it could be used equally well for measuring very low temperatures.

In some experiments made to determine what form of mouthpiece was best adapted to the tube, I found that the open tube sounded by blowing across the open end gave fewer overtones than when it was provided with a mouthpiece like an organ pipe.

I have thought that the melting point of very refractory substances could be determined by making the tube of the substance to be tested, and finding the highest pitch which it could be made to give before melting; but I have at present no source of sufficiently high temperature available for the test.

LELAND-STANFORD UNIVERSITY,
March, 1893.

METHOD FOR DETERMINING FOCAL LENGTHS OF MICROSCOPE LENSES.

W. S. FRANKLIN.

THE following is a method for determining the focal lengths of the lenses of a compound microscope *in situ*. It depends upon measurements of the magnifying power with short tube, the magnifying power with long tube, the change in the length of the tube, and the movement of the objective in refocussing. The method has been occasionally assigned by the author as a laboratory exercise since 1889.

Let m be the magnifying power of the microscope with short tube, and m' the magnifying power with long tube; magnifying power here signifying the ratio of apparent size of an object as seen with the microscope to its apparent size as seen with the naked eye at a distance of D cm.

Let p be the equivalent focal length of the eyepiece; p' the focal length of the objective; a the distance from the object to the center of the objective; and b the distance of the image from the center of the objective. Let k be the movement of the objective in refocussing, and l the change in the length of the tube. Let D be the distance of the auxiliary scale used in determining magnifying power.

Then.

$$m = \left(\frac{D}{p} + 1\right)\frac{b}{a}, \tag{1}$$

$$m' = \left(\frac{D}{p} + 1\right)\frac{b+l}{a-k}, \tag{2}$$

$$\frac{1}{p'} = \frac{1}{a} + \frac{1}{b} \tag{3}$$

$$\frac{1}{p'} = \frac{1}{a-k} + \frac{1}{b+l}, \tag{4}$$

From these equations we easily obtain:—

$$a^2lm - b^2km' = 0 \tag{5}$$

$$a^2 - \frac{2km'}{m'-m}a - \frac{klmm' - k^2m'^2}{(m'-m)^2} = 0. \tag{6}$$

Equation (6) enables the calculation of a. The value of b may then be determined from (5), and p and p' from (1) or (2) and (3) or (4) respectively.

The magnifying power is most easily determined by making use of an unsilvered slip of glass affixed to the eyepiece so as to enable an auxiliary scale to be seen in the field of the microscope. The power m will then be given as the apparent length on this scale of a unit length on a stage micrometer.

The distance, D, is to be taken as the distance from a certain one of the two principal, focal planes of the eyepiece, to the image (in the glass slip) of the auxiliary scale. In the case of the negative (Huyghens') eyepiece, D is the distance from a point midway between the two lenses to the image, in the glass slip, of the auxiliary scale. In case of the positive (Ramsden's) eyepiece this focal plane is between the two lenses of the combination at a distance from the eye lens equal to three-fourths of the distance between the lenses.

The change of length of the microscope tube may be easily measured directly. The movement k, in refocussing may be conveniently measured by means of a micrometer caliper mounted temporarily and playing against an arm fastened to the tube just above the objective.

The most serious source of error is that due to distortion unless, indeed, a very small object be used, when the determination of power will be inaccurate. However, errors due to distortion are of the same order of magnitude as the uncertainty in focal lengths due to spherical and chromatic aberration, and cannot in strictness be considered as detracting from the accuracy of the method.

The method has been carried out in the physical laboratory at this place upon a Beck microscope. A Brown and Sharpe micrometer caliper was used for measuring k. A Zeiss stage micrometer divided to $\frac{1}{10}$ mm. was used as the object, and l was measured directly. The distance D was chosen rather large so as to enable it to be measured with greater certainty.

The mean of three sets of observations gave $m = 184$, $m' = 289$, $k = .094$ cm., $l = 6.67$ cm., and $D = 88$ cm. Whence $a = 2.00$ cm., $b = 13.45$ cm., $p' = 1.74$ cm., $p = 3.33$ cm. With an objec-

tive of shorter focal length the mean of two sets of observations gave $p' = .507$ cm. and $p = 3.36$ cm. Duplicate sets of observations furnished values for p and p' which differed from the above by about one per cent.

AMES, IOWA, June 10, 1893.

SOME APPLICATIONS OF ELECTRIC HEATING IN PHYSICAL LABORATORY PRACTICE.

EDWARD L. NICHOLS.

IT is strange that so little attention has been given by physicists to the advantages of electric heating in calorimetric and other similar work. Professor Henry Crew[1] quite recently described "a new method of obtaining a constant temperature" in which a coil of wire is used to heat an air bath with admirable effect. It would seem that devices more or less closely related to the one due to his ingenuity must have occurred to others engaged in laboratory operations; but I recall no published account of such methods. The application of electric heating for temperature control has been in use in the physical laboratory of Cornell University for a considerable time, and it gives me pleasure to confirm Mr. Crew's statements as regards the excellent results which may be obtained by means of it.

It will be seen from what follows that the methods of procedure to be described differ from his chiefly in three particulars: (1) the use of naked wire, whereby the range of temperature attainable is much increased; (2) the absence of the ice bath, it having been found that remarkable constancy of temperature within a heated coil occurred, under almost all ordinary conditions of the surrounding space; (3) the measurement of temperatures by means of a calibrated coil of copper wire.

The method to be described consists in heating the body or apparatus in question by means of a spiral coil of uncovered wire surrounding it. The wire, which may be of German silver, is trav-

[1] Crew, Philosophical Magazine, 33 (5), p. 89, 1892.

ersed by strong electric currents, and thus brought to a high temperature. The best material for this purpose is some metal of high specific resistance and small temperature coefficient. It should be capable of withstanding high temperatures without melting. The method under consideration has been applied under my direction in a variety of laboratory experiments. I propose to describe several of these briefly for the purpose of illustrating its advantages.

I. The first of these experiments was suggested by a paper read by Professor William A. Rogers at the Cleveland meeting of the American Association for the Advancement of Science. He called attention in that paper to the fact that thermometers when subjected to a fall of temperature frequently indicate apparent discontinuity in the curve of cooling, the mercury column falling stepwise, and in some cases even retracing its course in the tube. This observation having been verified in the case of several thermometers of the best French and American manufacture, it seemed of interest to determine whether the phenomenon might not be due in part to some peculiarity in the law of the contraction of glass. The experiments were made by Mr. Ernest F. Nichols. Rods of glass were mounted upon the bed of a comparator. One end of the rod was fixed, and to the free end a mirror was attached in such a manner that the changes in the length of the rod would produce a motion of rotation. The image of a vertical scale set up at a distance of about three meters was observed by means of a reading telescope. In this way every change in the length of the rod was greatly magnified. The rod was surrounded by an open spiral coil of iron wire reaching from end to end. Sufficient current was sent through the wire to heat it, and this current, which was supplied by a storage battery, was kept constant until the movement of the image of the scale had ceased. The circuit was then broken, and the rod was allowed to cool. A curve of cooling was obtained by noting the movement of the image at frequent intervals until the rod had reached the temperature of the room again. Upon plotting this curve it was found that the discontinuities in the rate of cooling if present in the case of the specimen of glass tested were extremely

small. The curves were not perfectly true, but the irregularities were so small as to make it uncertain whether they were not to be ascribed to errors of observation or to imperfections in the apparatus. Rods of aluminium and brass were treated in the same manner for the purpose of determining whether these metals would give smoother curves of cooling than had been obtained in the case of glass. These curves also showed slight irregularities. The outcome of the experiment, therefore, was a negative one; but that it was evident from the curves obtained that the phenomenon noticed in thermometers could not be ascribed altogether to the suspected peculiarity in the law of the cooling of glass. From the point of view of the present paper, however, the experiment was of considerable interest. It made it evident that this means of heating is an advantageous one in the study of the linear dilation of rods of considerable length.

II. In the same year, 1891, the method was applied to a determination of the coefficient of expansion of a standard meter bar of brass. This experiment was carried on by Mr. Joseph Le Conte. For the purpose of determining the temperature of the bar the same was wrapped from end to end with several turns of fine silk-covered copper wire. The resistance of this longitudinal coil, the temperature coefficient of which had been previously determined with a good deal of care, was measured by the fall of potential method, using a sensitive galvanometer, a single cell of battery, and a compensated standard made by combining German silver and copper. Thus the temperature of the bar was averaged for its entire length. This bar was mounted upon a comparator, and was surrounded by a coil of large uncovered German silver wire in the manner described in Experiment I. The length of the bar was measured by means of reading microscopes attached to the comparator. The bed of the instrument was protected from the radiation of the coil by means of a layer of asbestos paper. It was found by trial that the rise of temperature of the massive iron bed of the comparator was almost unappreciable even when the coil was heated to a temperature of 50° C. In this experiment the measurements were taken with the circuit closed, the bar having been brought to the desired constant temperature

by long-continued application of the current. It was found that any temperature within wide range could be reached by simple adjustment of the resistance of the circuit through which the heating current flowed, and that no troublesome fluctuations took place. The coefficient of the bar was found to be in close agreement with determinations for this metal by the best known methods. It became evident in the course of the experiment that this method of applying heat in the determination of the coefficient of expansion of specimens of considerable size was very much more convenient and under much better control than any of those which have been used in the past. The method of measuring temperature by means of an integrating coil extending over the entire length of the bar was also found to be superior to any procedure involving the use of a mercury thermometer. How very difficult it is to place thermometers so as to obtain true indications of the temperature of a bar in process of being tested is well known to all who have attempted work of this description. The calibration of the coil of copper wire for temperature measurements showed that the temperature indicated by it represented very closely indeed the temperatures of the mass of metal which it surrounded. I shall have occasion to speak of this more in detail in the discussion of another experiment.

III. The next illustration of the use of this method of heating is taken from an investigation which has been in progress during the past year at the hands of Mr. J. S. Shearer. A detailed account of his experiments is now in preparation, and will be published shortly. Mr. Shearer studied the influence of temperature upon the rate of a tuning fork. In his work the method under consideration found double application. In the first place, for the purpose of measuring the rate of the fork while hot, he substituted for the rather cumbersome device employed by Kayser, in his well-known work upon this subject, the use of a spiral coil similar to those which have already been described. In this case, however, the fork was mounted vertically, and was driven by means of an electromagnetic contact device. The whole fork was surrounded by a cylindrical coil. This consisted of German silver wire wound upon a drum of asbestos. At the base of the fork other

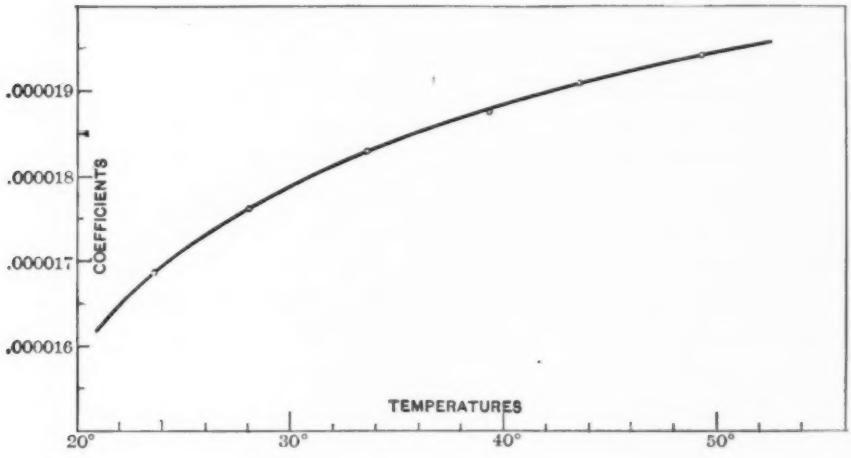
asbestos packing was used so as to prevent convection currents. It was found possible to bring the fork to any desired temperature up to 100° C., and to maintain it at that temperature for as long a time as one wished. The rate was determined by the method of Lissajous' figures. The distribution of temperatures in this case was less satisfactory than where the metal to be heated was mounted in a horizontal position, but the temperature differences within the coil were not sufficiently great to interfere seriously with the accuracy of the determinations. Here, again, the method of electric heating showed itself incomparably superior to those previously in vogue. Since the results are to be given elsewhere, I need only allude to those features of the experiment which touch upon the question before us. Having determined the law of change in the rate of vibration of the fork, it became important to ascertain in how far the change was due to the influences of temperature upon Young's modulus of elasticity. In previous work of this kind the values for Young's modulus, determined by experiments upon steel wires at the hands of Wertheim and others, had been made use of in the computation. The classical determinations of Wertheim for the elasticity of steel leave a point of great importance in the application of the theory of the vibration of tuning forks in doubt; some varieties showing the continued diminution in the value of the modulus, others, on the contrary, showing a rise in Young's modulus to a maximum with subsequent decrease. Under such circumstances it is not possible to make use of the data already at hand, and it becomes necessary to determine this constant in the case of the particular specimen of metal under investigation. By clamping the base of the fork securely to the bed of the comparator, and surrounding one prong with a spiral German silver coil, in the manner described in the first two experiments alluded to, it was found possible to get a very good determination of Young's modulus by the method of deflections, applying weights up to five kilograms to the free end of the fork, and noting the deflections by means of a mirror as in Experiment I. Changes in the modulus amounting to one per cent could have been detected with certainty. I do not know of any other means of heating a tuning fork and maintaining its tem-

perature constant during the time necessary to make such observations as these, which can be compared with ease of application and in nicety of control with the one which it is the purpose of this paper to describe; nor do I any method so convenient for determining the modulus of elasticity.

IV. An experiment illustrating this method of heating, in application to work of another character, is as follows: This is also taken from an investigation now in progress in the Physical Laboratory of Cornell University. It was desired to determine the effect of heating sugar solutions upon the rotation of the plane of polarization. In the Soleil saccharimeter the tube containing the fluid, the rotational coefficient of which was to be determined, was surrounded by a coil of German silver wire precisely in the method already indicated. It was found possible in this way to bring about temperature changes within the entire range below the boiling point of the solution, and to maintain them during the measurement of the angle of rotation. The entire apparatus consisted simply of a German silver helix, surrounding the brass tube of the saccharimeter, in series with suitable resistances, and a galvanometer for the measurement of current. Since the temperature itself could not be readily determined directly at the time of observation, a calibration test was made with the thermometer inserted in the liquid, the temperature being measured as a function of the current flowing through the coil. The relation between temperature and current having been ascertained once for all, it was therefore only necessary to note the readings of the galvanometer in order to find the temperature to which the measurement of the angle of rotation applied. A graduate student, Miss A. S. Baxter, has completed some preliminary experiments with this apparatus, and there is every reason to think that the method will lead to excellent results.

V. Another illustration of the use of this method occurs in some measurements similar to those of Mr. Le Conte (Experiment II.). In this case, however, the bar of metal to be studied was a piece of white speculum metal, one decimeter in length, ruled for use under the microscope. It was desired to determine the temperature coefficient, and the change of that coefficient with

change of temperature. Under my direction Mr. C. H. Sharp has recently completed such measurements. The piece of metal being short, it was not deemed necessary to integrate the temperatures by means of a copper coil. Thermometer readings, taken with the thermometer bulb in various positions, showed that the range of temperatures inside the heated coil was very small, and it seemed certain that the average of these readings represented quite accurately the temperature of the bar itself. The results obtained in this case are shown by means of the accompanying figure in which abscissæ are temperatures and ordinates are



coefficients of linear expansion corresponding thereto. It will be seen that the curve is not a straight line, but that, as has been shown by many early experimenters upon the expansion of solids, the coefficient increases notably with rise of temperature.

VI. My final example is drawn from some investigations upon the thermal conductivity of a copper bar. As the results will probably be described in this *Review*, I will merely say that the bar in question, which is one of those distributed by Professor W. A. Rogers in 1891 for the purpose, having been twice measured after established methods, was taken in hand by the present experimenters, Messrs. Child and Quick, who, in their final determinations, substituted for the bath at the hot end of

the bar a closely wound, snug-fitting coil of large wire carrying heavy currents. The control of this coil was found to be much more complete and very much simpler than that of any high temperature bath. Finally, to obtain the curve of cooling, the entire bar, one meter long, was surrounded with an open spiral and heated to nearly 300° C. It was found practicable to secure great uniformity of temperature for the entire length of the bar, no simple matter by the usual methods, to produce and maintain the heat without vitiating the surface, and finally to begin readings for the cooling curve immediately upon removal of the heating coil, thus extending the upward range of the measurements.

Temperatures were determined throughout the investigation by means of a closely fitting collar, consisting of one layer of smaller copper wire, the position of which upon the bar could be readily changed.

Enough has been stated in the account of these six experiments to indicate that electric heating has a wide application in the physical laboratory. To those who have occasion to direct or plan experimental work, the usefulness of such methods as have been described will be obvious.

PHYSICAL LABORATORY OF CORNELL UNIVERSITY,
June 23, 1893.

NEW BOOKS.

Electrical Papers. By OLIVER HEAVISIDE. 2 vols. London, Macmillan & Co., 1892.

The two volumes before us contain a reprint of fifty articles on electrical subjects, which appeared originally in *The English Mechanic*, *The Philosophical Magazine*, *The Journal of the Society of Telegraph Engineers*, *The Electrician*, *The Philosophical Transactions*, and *Nature* during the interval from July, 1872, to July, 1892. They contain in addition three papers which were written within that interval, but are now published for the first time. The first twelve articles are on matters dealing principally with telegraphy, and are but loosely connected. The next eight articles deal mainly with the theory of the propagation of variations of current along wires, beginning with applications of the simple electrostatic theory of Sir William Thomson to cables under different circumstances, and followed by extensions to include self-induction, or the influence of the inertia of the magnetic medium, and the mutual influence both electrostatic and magnetic of parallel wires. We next come to a series of papers relating to electrical theory in general, first published in *The Electrician* between 1882 and 1887; then to a study of the theory of the propagation of induction and electric current in round wires; and lastly to a more comprehensive treatment of electromagnetism, based upon Maxwell's theory in a long series of papers entitled "Electromagnetic Induction and its Propagation." The remaining articles consist of applications growing out of the preceding theory, namely, the self-induction of wires, resistance and conductance operators, electromagnetic waves, the forces, stresses, and fluxes of energy in the electromagnetic field, etc.

Thus it will be seen that the work, though a reprint of papers, has a continuity of treatment, and is suitable for the electrician or the advanced student, though not adapted for use as a manual. Mr. Heaviside is, I believe, now engaged putting the electromagnetic theory in a form suitable for a student.

One who reads these papers cannot fail to be impressed with the philosophic earnestness and philanthropic motive of the author. He is a lover of wisdom and a searcher after truth. In this connection it is interesting to read how some of his most important papers, namely, those on the self-induction of wires, were first received. Because his views were contradic-

tory of those then accepted by the post-office officials, an attempt was made to prevent their discussion and publication. These are the papers, which are now published for the first time. We cannot but agree with the author when he says "there seemed to be an idea that official views, in virtue of their official nature, should not be controverted or criticized. But there seems something wrong here, as the later evidence in support of my views has shown. For what other object have scientific men than to get at the truth, and how is it to be done without free discussion?"

The articles on electric and electromagnetic theory form the best commentary that has been written on Maxwell's "Electricity and Magnetism." The fundamental ideas are explained and elucidated, and the author is not content unless he places before the reader the true inwardness of the analytical expressions and equations. But they are more than a commentary: they constitute a development of Maxwell's theory.

In these volumes we have a full account of the author's system of rational electric units. In many cases the true relation between electric quantities is clouded by the appearance of an extraneous 4π; for example, the resultant force outwards at a charged surface is not equal to the density, but to 4π times the density. This is due to the manner in which the unit of electricity or the unit magnetic pole is defined. If the unit pole were defined as that from which unit amount of force emanated, the force at distance r would be $\frac{1}{4\pi r^2}$ instead of $\frac{1}{r^2}$. The advantage would be that 4π would appear only where its appearance is rational; all the equations would then be seen to be truths instead of appearing as mere formulæ. It is likely, therefore, that the use of rational units would be beneficial in theoretical investigations; but whether the practical units already defined should be made to change with every improvement in the theory of units is a different question.

The author makes (Vol. II., p. 201) some highly important observations on the relation of physical science to mathematical analysis. It will be, perhaps, in the recollection of some readers that Professor Sylvester, a few years since, in the course of his learned paper on Bipotential, poked fun at Professor Maxwell for having, in his investigation of the conjugate properties possessed by complete spherical-surface harmonics, made use of Green's theorem concerning the mutual energy of two electrified systems. He said (in effect, for the quotation is from memory) that one might as well prove the rule of three by the laws of hydrostatics, or something similar to that. In the second edition of his treatise, Professor Maxwell made some remarks that appear to be meant for a reply to this; to the effect that although names involving physical ideas are given to certain quantities, yet, as the reasoning is purely mathematical, the physicist has a right to assist himself by the physical ideas. Certainly; but there is much more in

it than that. For not only the conjugate properties of spherical harmonics, but those of all other functions of the fluctuating character which present themselves in physical problems, including the infinitely undiscoverable, are involved in the principle of energy, and are most simply and immediately proved by it, and predicted beforehand. We may indeed get rid of the principle of energy, and treat the matter as a question of the properties of quadratic functions; a method which may commend itself to the pure mathematician. But by the use of the principle of energy, and assisted by the physical ideas involved, we are enabled to go straight to the mark at once and avoid the unnecessary complexities connected with the use of the special functions in question, which may be so great as to wholly prevent the recognition of the properties which, through the principle of energy, are necessitated. It seems to me that we may go even further, and hold that mathematical analysis should be based not on arbitrary laws, but on physical principles. Certain it is that the great developments of analysis have been due to an infusion of physical ideas and principles.

The author goes much further than Clerk Maxwell in the use of space-analysis. The position of Clerk Maxwell was paradoxical; he considered that the notation of quaternions was good, but the method itself bad; he made use of the former, but not of the latter. Mr. Heaviside uses a method which he thus describes (Vol. II., p. 521): "As electromagnetism swarms with vectors, the proper language for its expression and investigation is the Algebra of Vectors. An account is therefore given of the method employed by the author for some years past. The quaternionic basis is rejected, and the algebra is based upon a few definitions of notation merely. It may be regarded as quaternions without quaternions, and simplified to the uttermost; or else as being merely a conveniently condensed expression of the Cartesian mathematics, understandable by all who are acquainted with Cartesian methods, and with which the vectorial algebra is made to harmonize. It is confidently recommended as a practical working system."

It strikes me that to base a method upon a few definitions of notation is a highly illogical procedure, and indeed cannot be done. Let us see how it is attempted. A vector is denoted by a Clarendon letter as **A**, its magnitude by the corresponding Italic capital as A, and the magnitudes of its three rectangular components by A_1, A_2, A_3. This is a matter of notation, and the notation is real, in the sense of corresponding to something in nature. But observe the next step. "The scalar product of a pair of vectors **A** and **B** is denoted by **AB**, and is defined to be

$$\mathbf{AB} = A_1B_1 + A_2B_2 + A_3B_3 = AB\cos\widehat{\mathbf{AB}} = \mathbf{BA}''.$$

This is not a definition merely. If we define $AB\cos\widehat{\mathbf{AB}}$ as the scalar product of **A** and **B**, and denote it by the simple juxtaposition of its factors,

then A_1B_1 is the scalar product of A_1i and B_1i; A_2B_2 is the scalar product of A_2j and B_2j, and A_3B_3 is the scalar product of A_3K and B_3K. That $AB\cos\widehat{\mathbf{AB}}$ is equal to $A_1B_1\cos ii + A_2B_2\cos jj + A_3B_3\cos kk$ is not a matter of definition, but is a geometrical truth which may be demonstrated. It is a principle, not a definition. The author's introduction of the vector product contains a similar confusion of definition and principles. He says, "The vector product of a pair of vectors is denoted by $V\mathbf{AB}$, and is defined to be the vector whose tensor is $AB\sin\widehat{\mathbf{AB}}$, and whose direction is perpendicular to the plane of **A** and **B**, thus

$$\mathbf{VAB} = i(A_2B_3 - A_3B_2) + j(A_3B_1 - A_1B_3) + k(A_1B_2 - A_2B_1)$$

where i, j, k are any three mutually rectangular unit vectors." Now the definition refers merely to the meaning of **V**, and applies to all the terms of the above equation; while the equation itself asserts the geometrical truth, that the directed area formed by **A** and **B** is equal to the geometrical sum of the three areas specified, Mr. Heaviside, in avoiding the Scylla of metaphysics, has been drawn into the Charybdis of definition. It seems to me that the true basis for the principles of space-analysis is to be found in geometry, and more generally in physical science.

Mr. Heaviside opposes the quaternionic practice of introducing a minus before the scalar product; he holds that

$$\mathbf{SAB} = A_1B_1 + A_2B_2 + A_3B_3,$$

while according to Hamilton

$$\mathbf{SAB} = -(A_1B_1 + A_2B_2 + A_3B_3).$$

The quaternionic practice is a consequence of making Nature more simple than she really is, namely, the identifying of vectors and versors. This principle logically carried out would land us in believing that there is no difference between direction and change of direction, or between a translation and a rotation.

The square of a vector being positive, it follows that the reciprocal of a vector has the same direction as the vector but the reciprocal magnitude. This also agrees much better with the principles of analysis than does the quarternionic doctrine that the direction of the reciprocal is the opposite of that of the vector. The author is fully impressed with the importance of preserving harmony with the Cartesian analysis — logical harmony, so that we may pass from the one to the other with as much ease as we pass from algebra to arithmetic.

Mr. Heaviside in these volumes displays some wit, and that may be the reason why he describes his practical working system as "Quaternions without quarternions." The phrase is suggestive of the play of Hamlet with the character of Hamlet left out. To keep the quarternion out of

the play he certainly resorts to extreme measures. For instance, in an early paper (Vol. I., p. 271) he gets a product which is the sum of a scalar product and a vector product; namely, if

$$\nabla = i\frac{d}{dx} + j\frac{d}{dy} + k\frac{d}{dr} \text{ and } \mathbf{R} = Xi + Yj + Zk,$$

then

$$\nabla\mathbf{R} = -\left(\frac{dX}{dx} + \frac{dY}{dy} + \frac{dZ}{dz}\right) + i\left(\frac{dZ}{dy} - \frac{dY}{dz}\right) + j\left(\frac{dX}{dz} - \frac{dZ}{dx}\right) + k\left(\frac{dY}{dx} - \frac{dX}{dy}\right).$$

In a foot-note now added he says: "As the above is the only paper in which I have used the quaternionic ideas and notation, it is perhaps desirable to emphasize the fact that the use was parenthetical. There is great advantage in most practical work in ignoring the quarternion altogether, and also the double signification of a vector above referred to, and in abolishing the quaternionic minus signs." But in the same paper (p. 269) we find the following admission: "Now it appears that when ∇ is applied to a vector, it gives its curl and its convergence respectively. This extraordinary effect of ∇ is not easily to be understood — although symbolically it works out very well, for there is undeniably a certain amount of mystery about the rules for vector multiplication."

The explanation which I have published of this mystery is, that the products of vectors and the products of versors are distinct, forming parts of analysis complementary to one another; that the *complete* product of two vectors consists of two partial products, the one scalar and the other vector; and that this complete product is a quaternion in the sense of being such a sum, but not in the original and proper sense of the word "the ratio of two vectors."

Physical analysis is not all a play of vectors, nor is it all a play of quarternions; there are many actors beside. Our author in revolting from the one extreme has gone to the other; but in securing fair play for the vectors he has furthered the development of that space-analysis which at the present time is the greatest want of the physicist and especially of the electrician.

ALEXANDER MACFARLANE.

Handbuch der Physiologischen Optik. H. VON HELMHOLTZ. 2d edition.

Since the appearance of the first edition of Helmholtz's classic work on physiological optics, investigation in this subject has brought out many new truths, and hence physicists will give a hearty reception to this new edition by the recognized authority on the subject. About two-thirds of the entire work has already appeared. The general arrangement of the book is about

the same as in the original edition; but several improvements are noticeable. All new paragraphs are distinguished by an *n* in the margin, and the double page numbering allows easy reference to be made to the original edition. The bibliography is very full, and the references are at the bottom of the page instead of at the end of the section, as was the case in the original.

The anatomical description of the eye is very complete, and exhibits the professor's early surgical training. The description of the retina is especially full, and includes an account of the recent work of Max Schultz and others. New paragraphs are also added under Dioptrics of the Eye, Accommodation, and Astigmatism. The most important additions, however, are on The Intensity of the Sensation of Light and Color-Blindness. Under the former heading there are fully seventy pages of new matter. It is principally devoted to luminosity (Helligkeit) in its various relations to color, retinal sensitiveness, persistence of vision, etc.

Much space is devoted to the subject of color-sensation and color-blindness. The author reviews the Young hypothesis of color-sensation as given in the first edition, points out the inconsistencies in it, and proposes a new modification. The new hypothesis postulates three primary color-sensations, as did the original Young-Helmholtz theory, but these elements are very different. They are carmine red, green of wave length about 560, and blue of wave length about 482, — and all much more highly saturated than any spectral colors. The author's new hypothesis to explain color-blindness supposes the two primary sensations of the dichroic eye to be produced by two colors derived from the three normal fundamental colors, but not necessarily identical with them. The method of the derivation of the dichroic colors from the normal fundamental is not indicated, and there seems no necessary manner of derivation. This new view accords better with the recent work of Koenig and Dieterici than the original Young-Helmholtz theory, from the fact that it requires no distinction of dichromatism into red-blindness and green-blindness.

The Hering theory is very clearly enunciated and compared with that of the author. This theory he considers as simply a modification of the Young theory, which explains the facts of color mixtures as well as, but not better than, the original. It differs principally in the choice of the elementary sensations. In the Hering theory there are three fundamental color-sensations of which at least two exhibit both positive and negative characteristics. One of them gives white under excitation, and black when at rest; another gives blue and yellow, and the third gives green and its complementary red. White light excites only the white-black visual component; yellow excites this and also the blue-yellow component. Blue does the same, but in the opposite sense. When the blue and the green

excitations are in equilibrium, they exert no effect on the blue-yellow component. Also, according to this theory the sensation of luminosity depends solely upon the excitation of the white-black component.

E. S. FERRY.

Matter, Ether, and Motion. By A. E. DOLBEAR. 8vo, pp. 334. Boston, Lee and Shepard, 1892.

This little book is written for the use of the "large body of persons desirous of knowing more of natural philosophy, and especially of its trend, who have neither time nor opportunity to read and digest monographs on a thousand topics." The book will be found to treat of the whole domain of physics as fully as could be expected in so small a volume, and abounds in mechanical analogies and numerical illustrations. The necessity of the ether in modern physical theories is strongly emphasized, while the views of the author on the vortex atom theory are indicated by the remarks (page 43) that "there is no other theory that has any degree of probability at all."

Hurried writing and proof-reading are doubtless to blame for the misprints and inaccurate statements that occur at frequent intervals throughout the book. In most cases these errors are merely a source of annoyance, but with readers who are unfamiliar with the subject they might often be the cause of serious misconceptions. As an example of the numerous misprints may be mentioned the omission of the word "thousand" on pages 80 and 81, which makes it appear that the velocity of light is only 185 miles per second. The occurrence of this error on two consecutive pages can scarcely fail to prove confusing. At the top of page 64 will be found a numerical error which cannot be explained by a single misprint. In fact, I have been unable to discover how the result, which is wrong, could have been obtained by any ordinary mistake. Accuracy in numerical work in a book of this kind is doubtless of secondary importance; yet numerical illustrations constitute so large a part of the contents that it seems essential that the reader's confidence in the results should not be shaken. An error of a more serious character appears in the discussion of the thickness of a soap film on page 11, where the interference effects are explained with no mention of the change of phase introduced by reflection. A result is thus obtained which is half what it should be. The statement made on page 68 that a weight resting upon a table requires energy for its support is extremely unfortunate. In this, as in several other cases, the explanation is so brief that many will fail to appreciate the author's real meaning, and without suitable interpretation the statement is certainly in the highest degree misleading.

There will be noticed throughout the book an indiscriminant mingling of well established theory and more or less plausible speculation, with no means by which the beginner can distinguish between then. This renders the book to a certain extent unsuitable for the class of readers for whom it is intended. But the numerous mechanical analogies, which in many cases are extremely ingenious, together with the easy conversational style in which the book is written, will make it interesting and suggestive reading for those who are well grounded in physical principles.

ERNEST MERRITT.

Recent Text-books in Physics.[1] — Mr. Selby's text-book on Mechanics is intended for students who are familiar with the elements of geometry and algebra, but whose mathematical preparation does not extend beyond this point. In addition to a discussion of the mechanics of the particle the book treats of the motion of an extended body, and also contains chapters on Gravitation, Elasticity, Hydrostatics, and Capillarity. In spite of the restrictions imposed by the necessity of elementary mathematical methods, the treatment in the earlier portions of the book is highly satisfactory, and it is not until the chapters on Gravitation and Elasticity are reached that the complexity of the proofs becomes annoying. A commendable feature of the book is the frequent insertion of numerical problems. The use of the pound as the unit of mass may cause some confusion to engineering students in this country, but the employment of the various systems of units is in all cases perfectly consistent.

In the new edition of the well-known laboratory manual of Glazebrook and Shaw no change has been made in the general method of treatment. Important additions, usually in the form of new practise experiments, have however been introduced throughout the book, the result being an increase of almost 150 pages over the edition of 1889.

In the chapters on Mechanics new experiments will be noticed, dealing with falling bodies, the fly-wheel, the ballistic pendulum, graphical statics, and capillarity. The chapter on Magnetism has been enlarged by the addition of four experiments, the most important of these being upon the study of magnetic induction by means of the magnetometer. The ballistic method, so common in this country, has unfortunately not been mentioned,

[1] *Elementary Mechanics of Solids and Fluids.* By A. L. SELBY. 8vo, pp. 299. Oxford, Clarendon Press, 1893.

Practical Physics. By R. T. GLAZEBROOK and W. N. SHAW. Fourth edition. 8vo, pp. 633. London, Longmans, Green & Co., 1893.

Lehrbuch der Experimentalphysik. By E. VON LOMMEL. 8vo, pp. 643. Leipzig, Johann Ambrosius Barth, 1893.

although a satisfactory discussion of the ballistic galvanometer itself is given in connection with the subject of capacity measurements. Among the additions to the chapters on Electricity may be mentioned experiments on electromagnetic induction, the silver voltameter, the use of the Clark cell, and the measurement of coefficients of induction. Two experiments have been added to the chapter on Calorimetry, and three to the chapters on Light. In fact, new matter will be found in almost every part of the book.

It is doubtless unnecessary to refer to the many excellent features of a book which is familiar to all teachers of physics. Suffice it to say that the matter that has been added is presented with the same care and clearness that have characterized the earlier editions.

Professor Lommel's text-book, while intended for the use of students desiring to review the subject of Experimental Physics, is yet arranged in such form as to be available as a text-book for beginners. The style is direct and clear, and illustrated diagrams are numerous. So far as possible the arrangement of matter has been made to correspond with the historical development of the science, and the names of the principal investigators have been mentioned in connection with each phenomenon described.

As is usual in the case of elementary text-books, the mathematical treatment suffers from the necessity of avoiding calculus methods. In cases where the subject cannot be properly treated without the use of the calculus Professor Lommel has, however, omitted the mathematical discussion altogether, and has thus avoided most of the unsatisfactory proofs which encumber so many of our text-books. The only departure from this rule which I have noticed is in the discussion of the value of the potential at a point, on page 278. The proof there given of the fact that the potential is universally proportional to the distance from the acting mass, although common, is by no means rigorous.

The value of Professor Lommel's book is increased by the explanations that are frequently given of recent developments in experimental physics. Thus there will be found a brief but intelligible statement of Lippmann's method of color photography, and of Wiener's experiments in stationary light waves. The principle of the "drehstrom" motor is also explained. The distinctly modern character of the work, together with the general excellence of the methods of presentation, make it a matter of especial regret that the book will not be generally available for American students.

ERNEST MERRITT.